

Theoretical and Observational Confrontation of Cosmology and Gravity, and the New Era of Multi-messenger Astronomy

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Goal

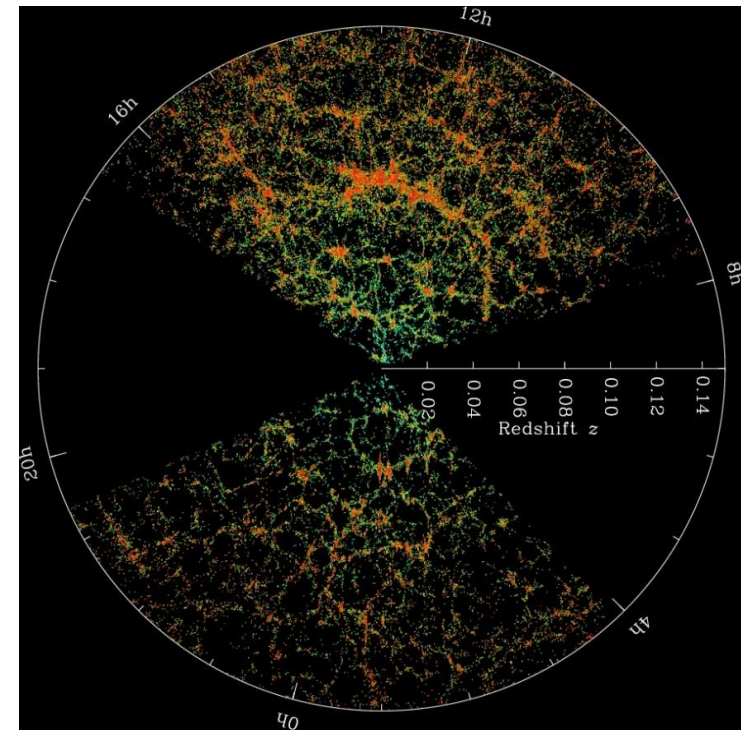
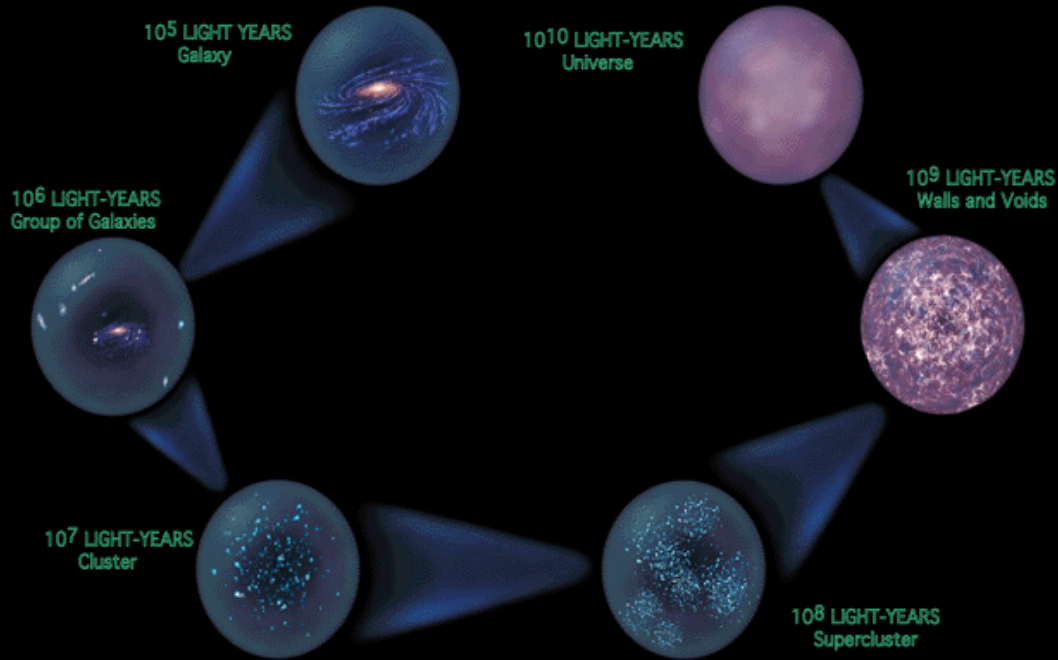
- We investigate **cosmological scenarios** that can describe the **observed Universe** as a whole
- **Astrophysical cosmology** has become a **precision science** with a huge amount of data. The advancing **gravitational wave multi-messenger astronomy** opens a **new era**



Talk Plan

- 1) Observational Cosmology: the **Standard Model of Cosmology**.
- 2) **Standard Model of Cosmology**. Do we need **new physics**?
- 3) We can **modify** the **Universe content**, or/and the **gravitational theory**.
- 4) Use of various **observational data** (SnIa, CMB, BAO, H(z), LSS etc) in order to **constrain** the proposed **theories**.
- 5) **GWs**: basic **properties** and **evolution**.
- 6) **Gravitational wave astronomy**, and **multi-messenger astronomy**: a **novel tool** to test General Relativity and cosmological scenarios in **great accuracy**.

Observations



- **SDSS** (Sloan Digital Sky Survey) 2004: \sim clusters "above and below the galactic plane" up to 1 Gpc



Observations

- As the scale we observe the Universe increases, it looks as homogeneous and isotropic.
- **Cosmological Principle**: “axiom” (indirect result)
 - I) We know that earth is an **isotropic** observation point.
 - II) An anisotropic system has up to one isotropic observation point.
- Hence, either we lie in the **only isotropic observation** point in an anisotropic Universe, or **all its points are isotropic** observation points.
- Thus, the Universe is **homogeneous and isotropic** (isotropic and inhomogeneous is not possible)

Observations

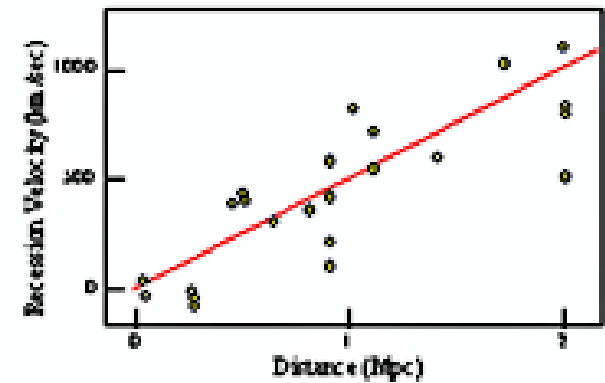
- Hubble 1929: The Universe expands



Hubble excelled in every course at school (except spelling), but was better known for his athletic prowess. He was a star player in football, baseball, and basketball, and ran track in high school and at the University of Chicago, where he earned a Bachelor of Science in 1910.



Hubble's Data (1929)



$$v = H r$$

$$H_0 \approx 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

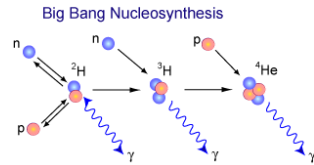


Observations

- Since the Universe expands it is reasonable that it **originates** from a “too tiny” and “too dense” “**primordial atom**” (Lemaitre 1927)
- Alpher, Bethe, Gamow (1948): The Universe **begun to expand** from a very **high-density and high-temperature** state towards less dense and hot states. Hoyle named the theory “**The Big Bang Theory**”.
- **Prediction I:** **Nucleosynthesis** has **primordial** origin, namely at first 3 minutes ($\sim 10^9$ K) (giving 25% Helium) and not in stars (1-4%)
As observed.

Observations

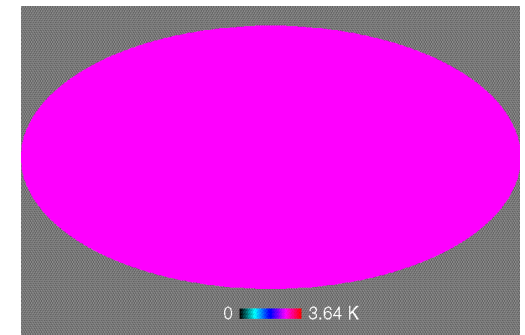
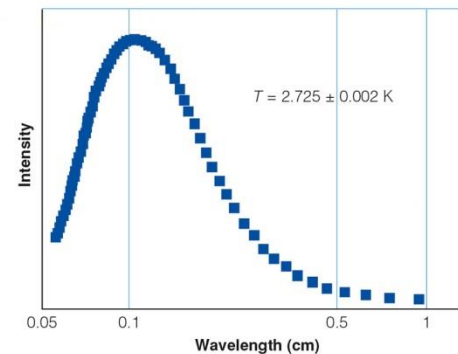
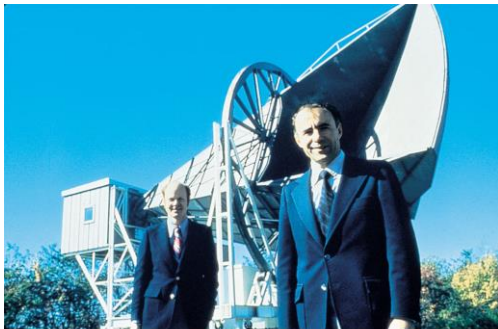
- **Prediction II:** The primordial Universe became full of high-energy photons



$$\lambda \approx 7 \cdot 10^{-12} \text{ cm},$$

380.000 years after ($\sim 3000\text{K}$) they decouple from electrons (Recombination era). Black body radiation (today $\sim 2.7 \text{ K}$)

- 1965 Penzias και Wilson





Theoretical arguments

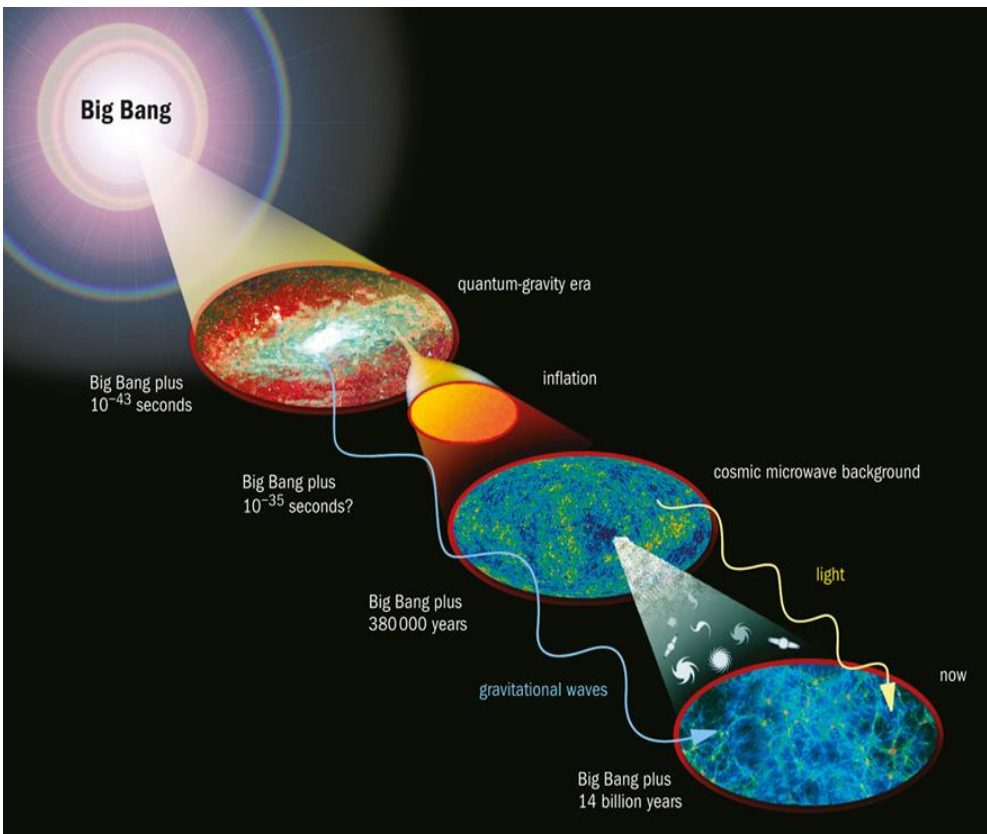
- Big Bang Theory explained: **Olbers paradox** (1826) (why night sky is not bright), **Ryle** (1970) (Radio galaxies density increases with redshift), **Element abundance**, **CMB**, etc
- **Theoretical Problems:**
 - I) **Horizon problem**: Why points at opposite directions have the same properties
 - II) **Flatness problem**: Why the universe is today almost spatially flat $\Omega_k \sim 0.001$. It must have started with $\sim 10^{-50}$!
 - **Monopole problem**: They are not observed.



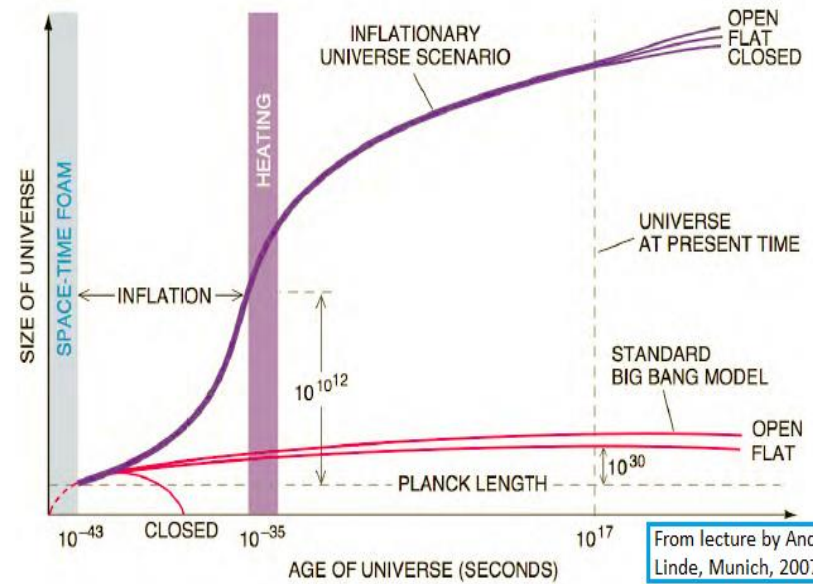
Inflation

- Kazanas, Guth, Linde (1982): The Universe 10^{-36} sec after the Big Bang, through some mechanism went into an exponential expansion up to 10^{-32} sec increasing in size $\sim 10^{30}$ times: Inflation.
- I) The observable Universe is a tiny part of the total one, and originates from a small, causally connected region.
- II) Due to the huge expansion, the spatial curvature became almost zero.
- III) Due to the huge expansion the monopoles spread in all regions, and thus our own, observable universe, has at most one.

Inflation

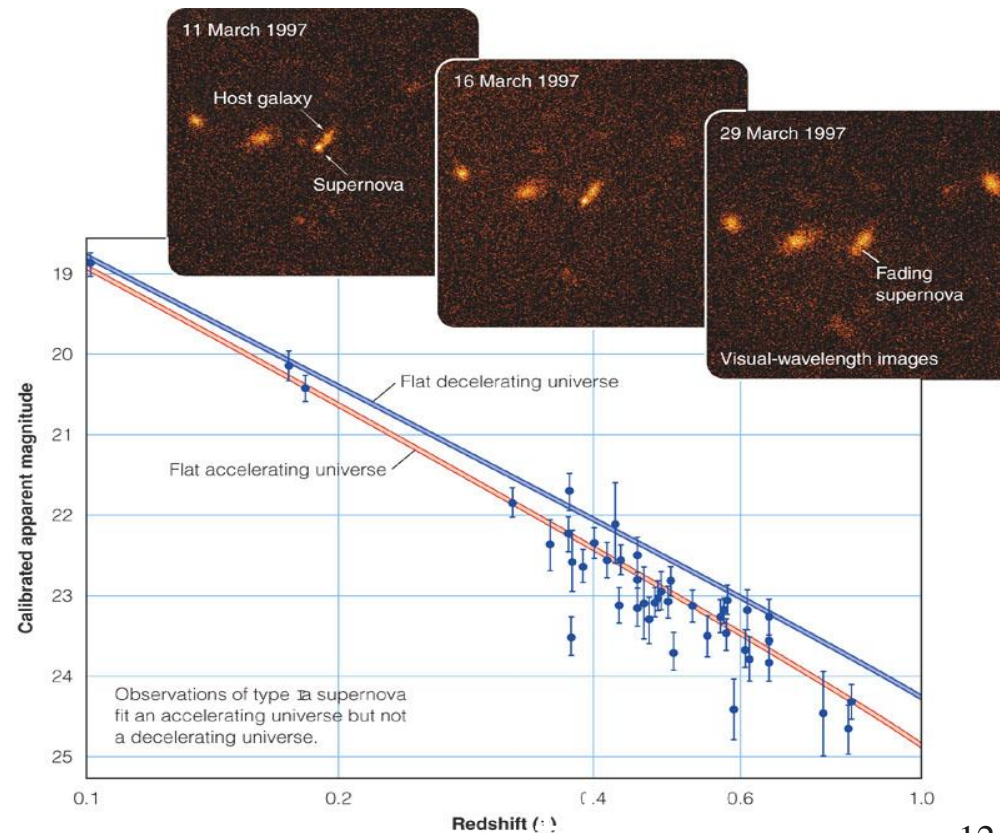


Inflationary Universe



Dark Energy

- The **Supernovae type Ia** (explosions of binaries with one being white dwarf) are “**standard candles**”, since their absolute magnitude M can be determined.
- In 1998, Perlmutter, Schmidt, Riess observed that 50 SNIa had **smaller apparent magnitude** than expected hence **light traveled more**, and thus the Universe **today expands faster** than before!



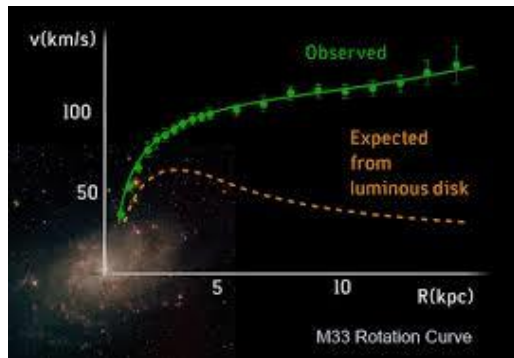


Dark Energy

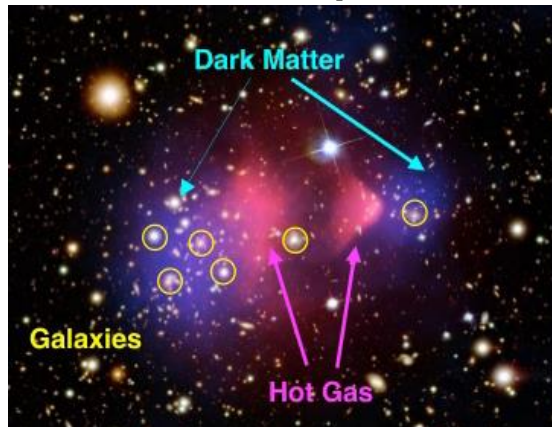
- The **accelerated expansion** is verified by independent observations, **Cosmic Microwave Background (CMB)**, **Baryon Acoustic Oscillations (BAO)**, **Large Scale Structure (LSS)**, etc
- Around **70%** of the **total energy density** of the Universe is this unknown **dark energy** (it does not interact electromagnetically).
- Possible explanation: **The cosmological constant Λ** (**Einstein's "greatest blunder"**). A term that produces the extra **"repulsion"**.

Dark Matter

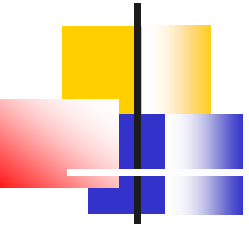
- Galaxy rotation curves:



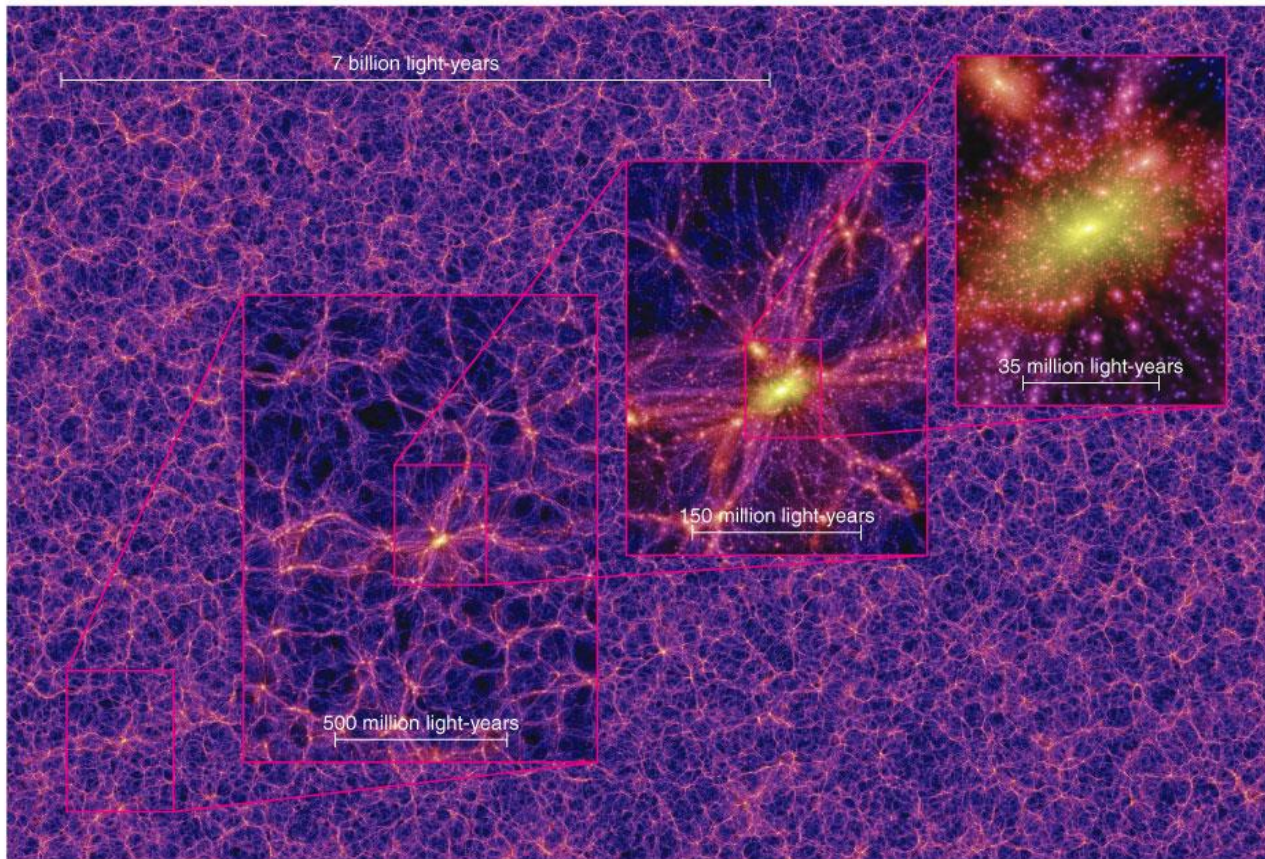
- Bullet cluster (collision of two galaxy clusters)



- 80% of matter is an "unknown" dark matter (it does not interact electromagnetically)!

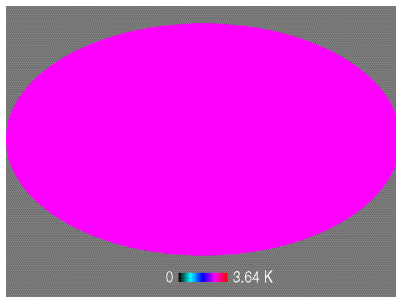


Dark Matter

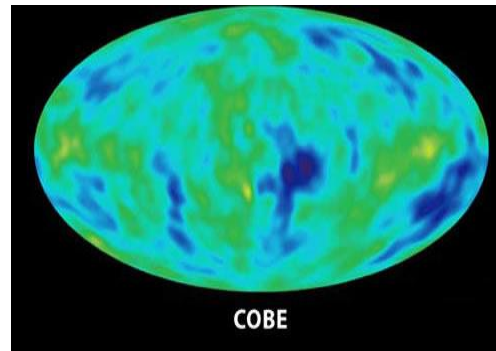


Cosmic Microwave Background radiation

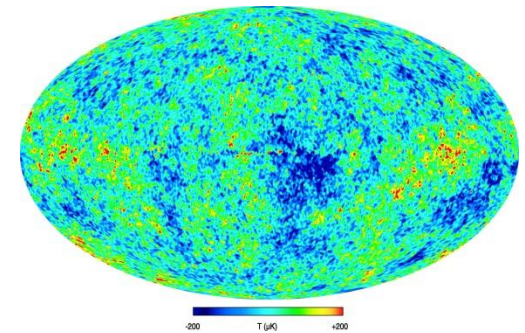
- Since 1989, COBE, WMAP and Planck satellites show that CMB has small **fluctuations**:



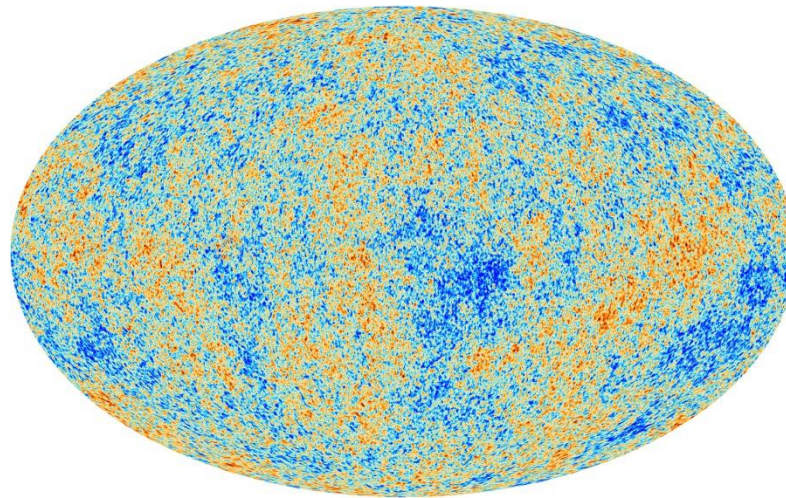
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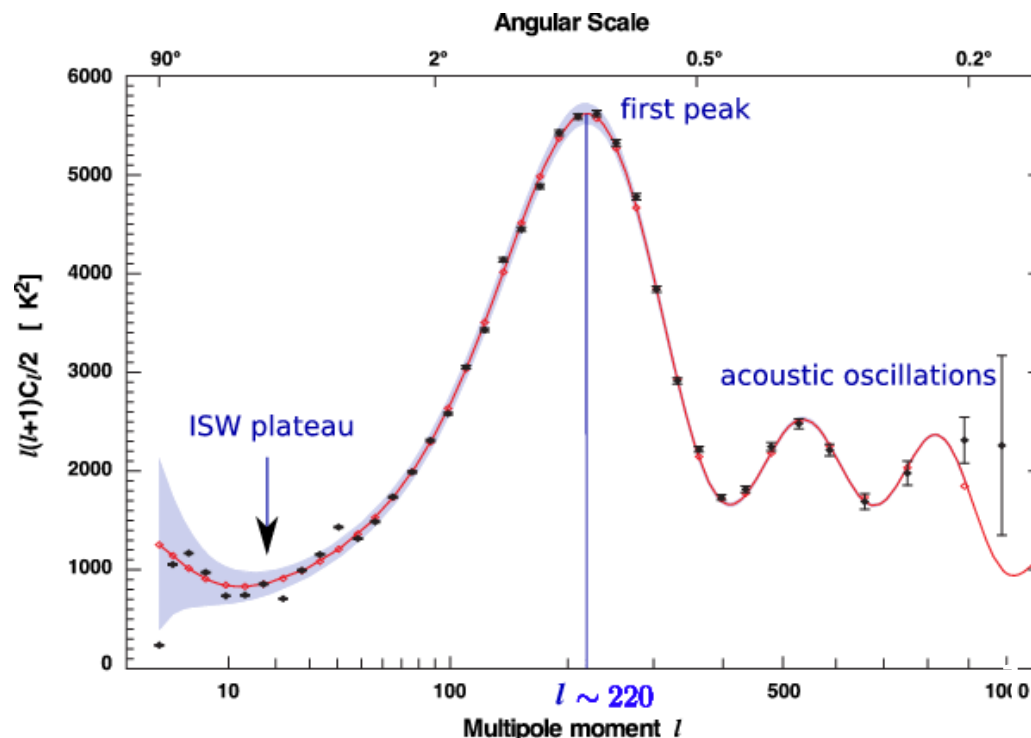


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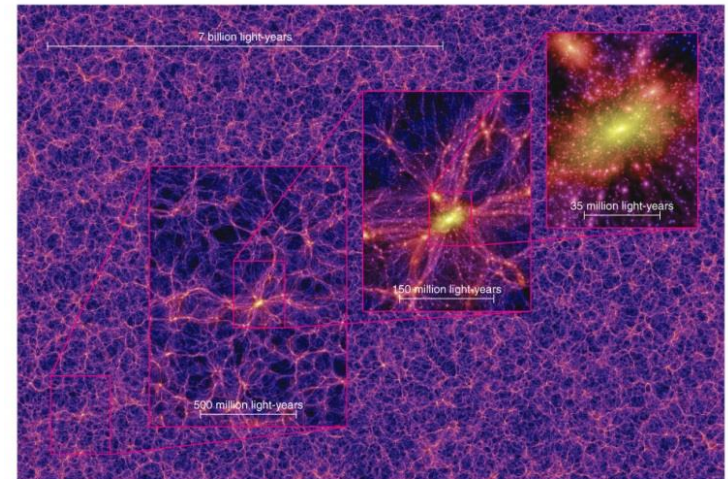
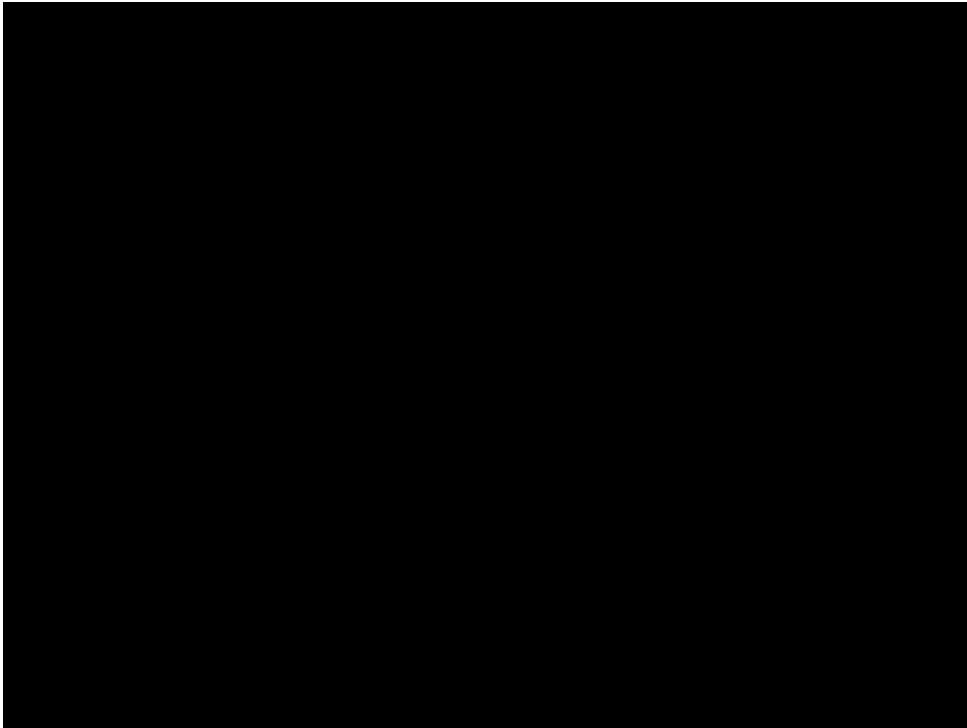
Cosmic Microwave Background radiation

- From the **fluctuation spectrum** we extract information: The **first peak** provides the spatial **curvature** (it results to flat universe), the **second peak** the **baryon energy density parameter**, the **third peak** the **dark matter energy density parameter**, etc.



Inflation can also explain CMB and seeds of LSS

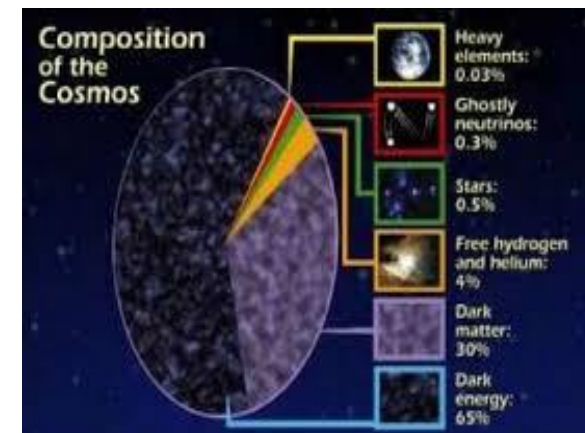
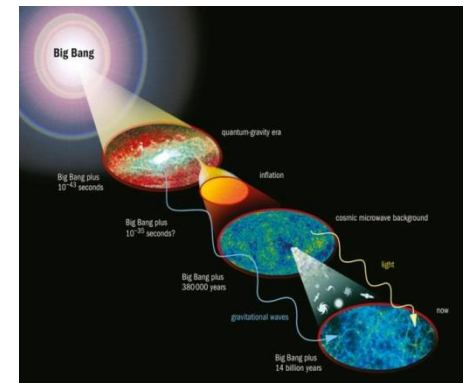
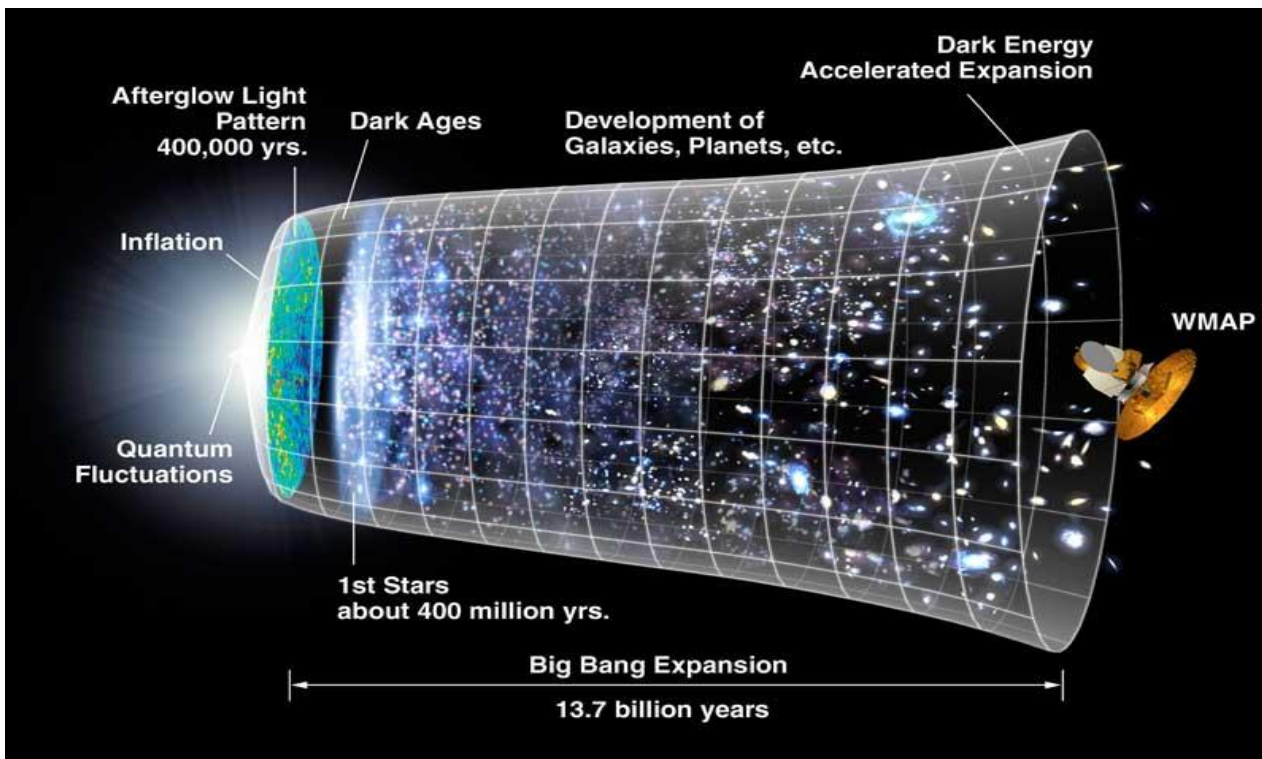
- Additional success: **Inflation** provides the necessary **primordial fluctuations**, which later gave the **Large Scale Structure** of matter:



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Summary of Observations

The **Universe history:**



Knowledge of Physics

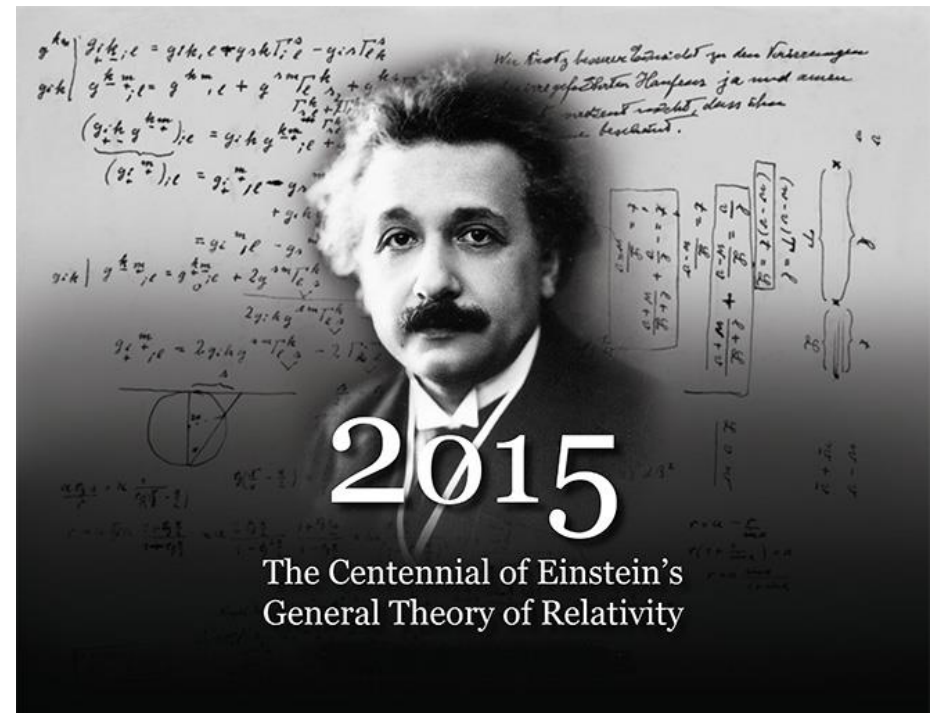
Knowledge of Physics: Standard Model

	<p>mass → $\approx 2.3 \text{ MeV}/c^2$</p> <p>charge → $2/3$</p> <p>spin → $1/2$</p>	<p>mass → $\approx 1.275 \text{ GeV}/c^2$</p> <p>charge → $2/3$</p> <p>spin → $1/2$</p>	<p>mass → $\approx 173.07 \text{ GeV}/c^2$</p> <p>charge → $2/3$</p> <p>spin → $1/2$</p>	<p>mass → 0</p> <p>charge → 0</p> <p>spin → 1</p>	<p>mass → $\approx 126 \text{ GeV}/c^2$</p> <p>charge → 0</p> <p>spin → 0</p>
	u up	c charm	t top	g gluon	H Higgs boson
QUARKS	<p>mass → $\approx 4.8 \text{ MeV}/c^2$</p> <p>charge → $-1/3$</p> <p>spin → $1/2$</p>	<p>mass → $\approx 95 \text{ MeV}/c^2$</p> <p>charge → $-1/3$</p> <p>spin → $1/2$</p>	<p>mass → $\approx 4.18 \text{ GeV}/c^2$</p> <p>charge → $-1/3$</p> <p>spin → $1/2$</p>	<p>mass → 0</p> <p>charge → 0</p> <p>spin → 1</p>	
	d down	s strange	b bottom	γ photon	
	<p>mass → $0.511 \text{ MeV}/c^2$</p> <p>charge → -1</p> <p>spin → $1/2$</p>	<p>mass → $105.7 \text{ MeV}/c^2$</p> <p>charge → -1</p> <p>spin → $1/2$</p>	<p>mass → $1.777 \text{ GeV}/c^2$</p> <p>charge → -1</p> <p>spin → $1/2$</p>	<p>mass → $91.2 \text{ GeV}/c^2$</p> <p>charge → 0</p> <p>spin → 1</p>	
	e electron	μ muon	τ tau	Z Z boson	
LEPTONS	<p>mass → $< 2.2 \text{ eV}/c^2$</p> <p>charge → 0</p> <p>spin → $1/2$</p>	<p>mass → $< 0.17 \text{ MeV}/c^2$</p> <p>charge → 0</p> <p>spin → $1/2$</p>	<p>mass → $< 15.5 \text{ MeV}/c^2$</p> <p>charge → 0</p> <p>spin → $1/2$</p>	<p>mass → $80.4 \text{ GeV}/c^2$</p> <p>charge → ± 1</p> <p>spin → 1</p>	GAUGE BOSONS
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

Knowledge of Physics

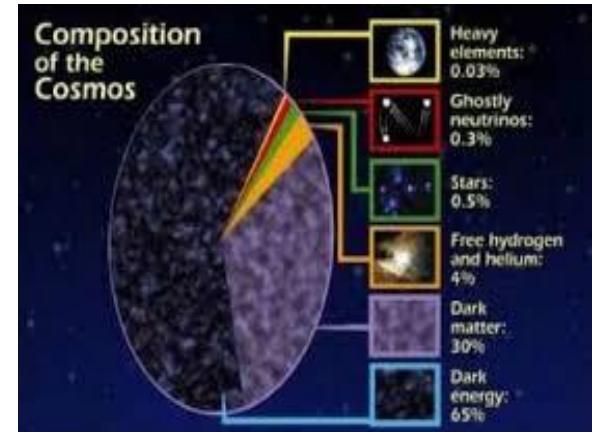
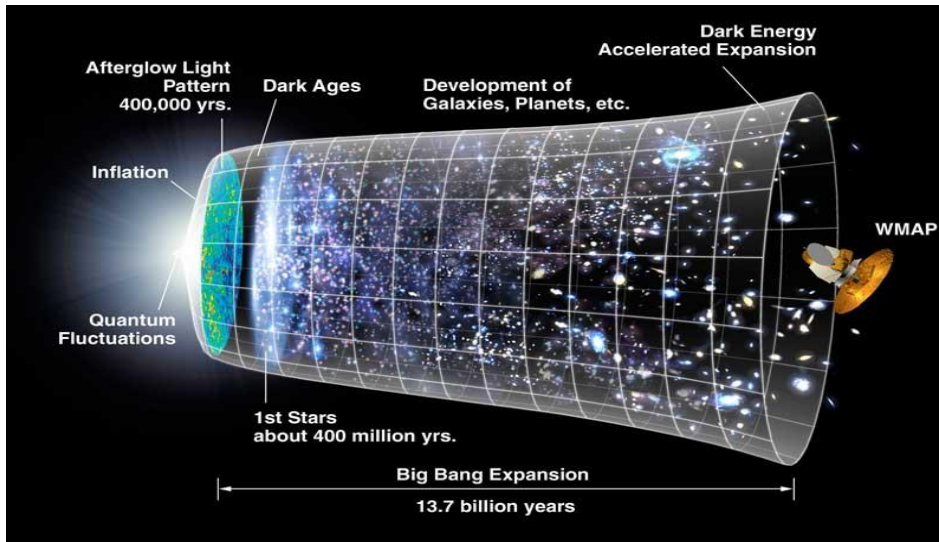
Knowledge of Physics: **Standard Model** + **General Relativity**

	<p>mass → $\approx 2.3 \text{ MeV}/c^2$</p> <p>charge → $2/3$</p> <p>spin → $1/2$</p> <p>u up</p>	<p>mass → $\approx 1.275 \text{ GeV}/c^2$</p> <p>charge → $2/3$</p> <p>spin → $1/2$</p> <p>c charm</p>	<p>mass → $\approx 173.07 \text{ GeV}/c^2$</p> <p>charge → $2/3$</p> <p>spin → $1/2$</p> <p>t top</p>	<p>mass → 0</p> <p>charge → 0</p> <p>spin → 1</p> <p>g gluon</p>	<p>mass → $\approx 126 \text{ GeV}/c^2$</p> <p>charge → 0</p> <p>spin → 0</p> <p>H Higgs boson</p>	
QUARKS	<p>mass → $\approx 4.8 \text{ MeV}/c^2$</p> <p>charge → $-1/3$</p> <p>spin → $1/2$</p> <p>d down</p>	<p>mass → $\approx 95 \text{ MeV}/c^2$</p> <p>charge → $-1/3$</p> <p>spin → $1/2$</p> <p>s strange</p>	<p>mass → $\approx 4.18 \text{ GeV}/c^2$</p> <p>charge → $-1/3$</p> <p>spin → $1/2$</p> <p>b bottom</p>	<p>mass → 0</p> <p>charge → 0</p> <p>spin → 1</p> <p>γ photon</p>		
	<p>mass → $0.511 \text{ MeV}/c^2$</p> <p>charge → -1</p> <p>spin → $1/2$</p> <p>e electron</p>	<p>mass → $105.7 \text{ MeV}/c^2$</p> <p>charge → -1</p> <p>spin → $1/2$</p> <p>μ muon</p>	<p>mass → $1.777 \text{ GeV}/c^2$</p> <p>charge → -1</p> <p>spin → $1/2$</p> <p>τ tau</p>	<p>mass → $91.2 \text{ GeV}/c^2$</p> <p>charge → 0</p> <p>spin → 1</p> <p>Z Z boson</p>	GAUGE BOSONS	
	<p>mass → $< 2 \text{ eV}/c^2$</p> <p>charge → 0</p> <p>spin → $1/2$</p> <p>ν_e electron neutrino</p>	<p>mass → $< 0.17 \text{ MeV}/c^2$</p> <p>charge → 0</p> <p>spin → $1/2$</p> <p>ν_μ muon neutrino</p>	<p>mass → $< 15.5 \text{ MeV}/c^2$</p> <p>charge → 0</p> <p>spin → $1/2$</p> <p>ν_τ tau neutrino</p>	<p>mass → $80.4 \text{ GeV}/c^2$</p> <p>charge → ± 1</p> <p>spin → 1</p> <p>W W boson</p>		
LEPTONS						



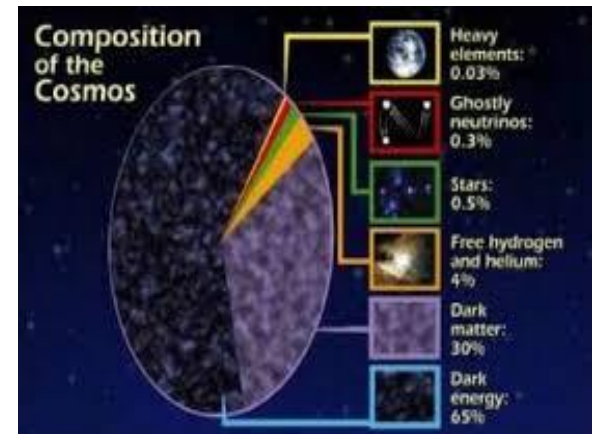
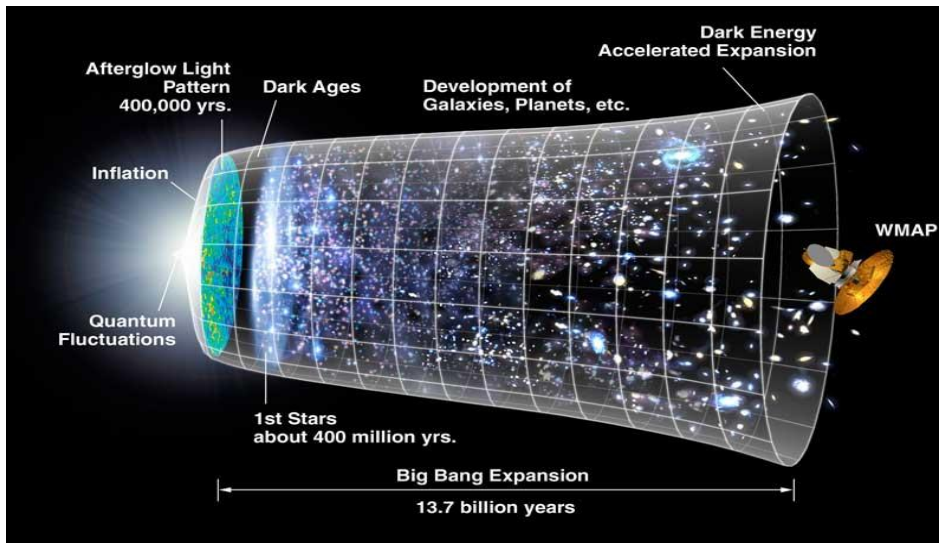
Modified/new knowledge of physics

So can our **knowledge of Physics** describes all these?



Modified/new knowledge of physics

So can our **knowledge of Physics** describes all these?



Most probably, no!

We definitely need **new physics** for **Inflation** and **Dark matter**. Maybe for **dark energy**.



Cosmology

- A **successful** cosmological model must:
 - 1) Describe the **evolution** of the universe at the **background level**
 - 2) Describe the **evolution** of the universe at the **perturbation level**



Cosmology

- A **successful cosmological model** must:
 - 1) Describe the **evolution** of the universe at the **background level**
 - 2) Describe the **evolution** of the universe at the **perturbation level**
- **Λ CDM paradigm** seems to succeed in **both**, at **post-inflationary** eras
- **Open issues:**
 - 1) The **cosmological-constant problem**. Calculation of Λ gives a number **120 orders of magnitude larger** than observed.
Worst error in the ~~history of physics, history of science, history~~
 - 2) How to describe **primordial universe** (inflation)
 - 3) **Tensions** with some data sets, e.g. **H_0 and $f\sigma_8$** data



Cosmology-background

- Homogeneity and isotropy: $ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$
- Background evolution (Friedmann equations) in flat space

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_{DE})$$

$$\dot{H} = -4\pi G (\rho_m + p_m + \rho_{DE} + p_{DE}),$$

(the effective DE sector can be either Λ or any possible modification)

- One must obtain a $H(z)$ and $\Omega_m(z)$ and $w_{DE}(z)$ in agreement with observations (SNIa, BAO, CMB shift parameter, $H(z)$ etc)



Cosmology-perturbations

- **Perturbation evolution:** $\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}} \rho \delta \approx 0$ where $\delta \equiv \delta\rho/\rho$
 where $G_{\text{eff}}(z, k)$ is the **effective Newton's constant**, given by

$$\nabla^2 \phi \approx 4\pi G_{\text{eff}} \rho \delta,$$

under the scalar **metric perturbation** $ds^2 = -(1 + 2\phi)dt^2 + a^2(1 - 2\psi)d\vec{x}^2$

- Hence: $\delta'' + \left(\frac{(H^2)'}{2H^2} - \frac{1}{1+z} \right) \delta' \approx \frac{3}{2}(1+z) \frac{H_0^2}{H^2} \frac{G_{\text{eff}}(z, k)}{G_N} \Omega_{0m} \delta$

with $f(a) = \frac{d \ln \delta}{d \ln a}$ the **growth rate**, with $f(a) = \Omega_m(a)^{\gamma(a)}$ and $\Omega_m(a) \equiv \frac{\Omega_{0m} a^{-3}}{H(a)^2/H_0^2}$

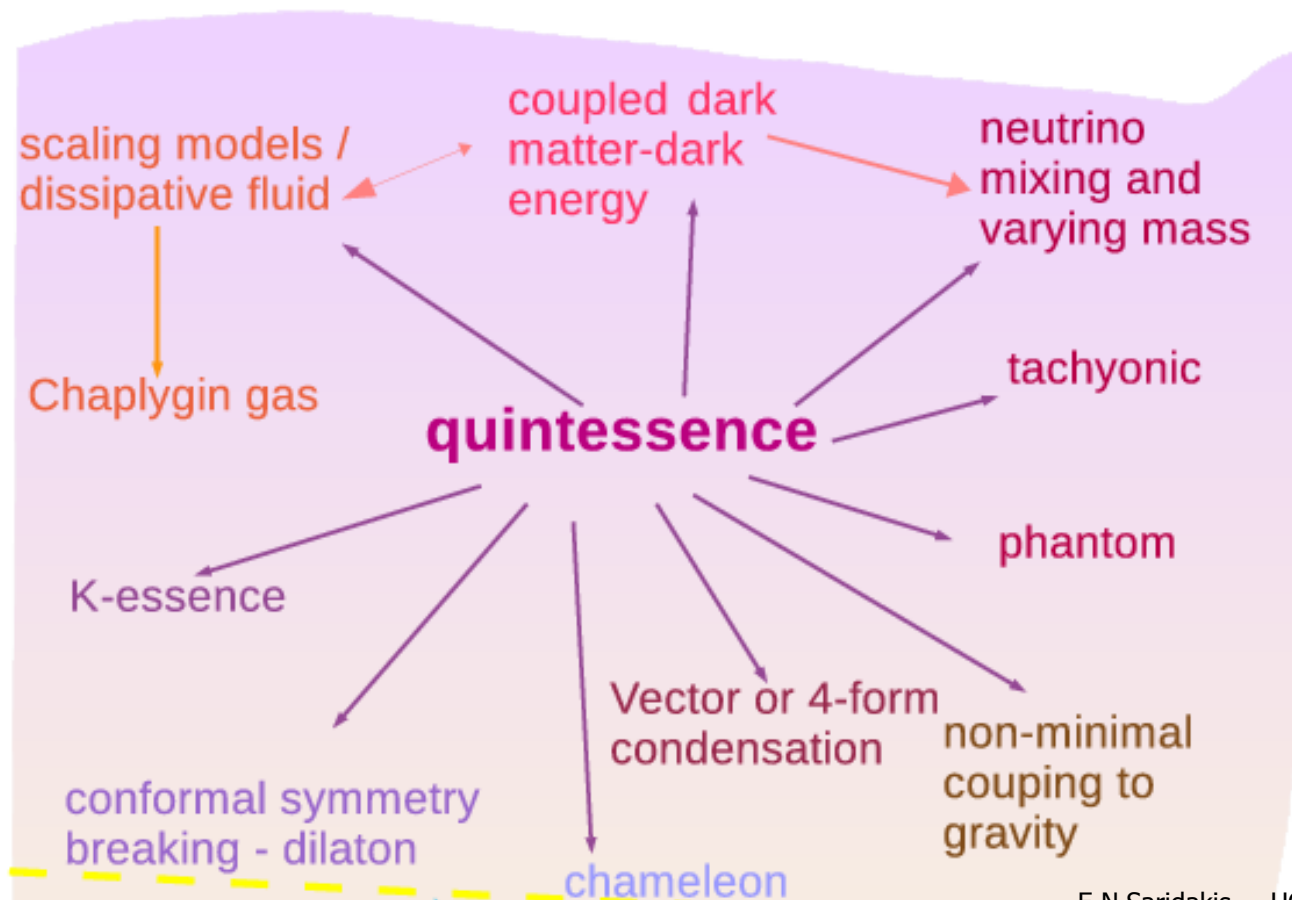
- One can define the **observable**: $f\sigma_8(a) \equiv f(a) \cdot \sigma(a) = \frac{\sigma_8}{\delta(1)} a \delta'(a)$

with $\sigma(a) = \sigma_8 \frac{\delta(a)}{\delta(1)}$ the z-dependent rms fluctuations of the linear density field within spheres of radius $R = 8h^{-1}\text{Mpc}$, and σ_8 its value today.

Dark Energy-Inflation

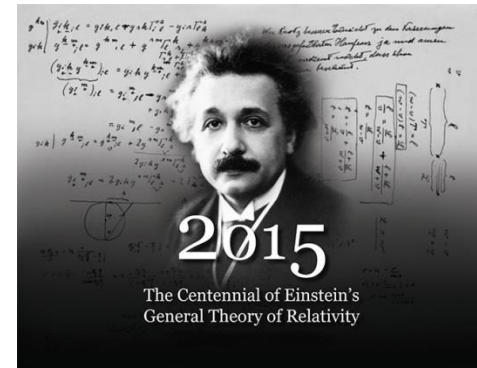
- Add a **scalar field ϕ** in the Universe content

mass = 4.2 MeV/c ² charge = 2/3	mass = 1.275 GeV/c ²	mass = 173.2 GeV/c ²	0	126 GeV/c ²
u up	c charm	t top	g gluon	H Higgs boson
mass = 4.8 MeV/c ² charge = -1/3	mass = 493 MeV/c ²	mass = 4.18 GeV/c ²	0	
d down	s strange	b bottom	γ photon	
mass = 0.511 MeV/c ² charge = -1	mass = 105.7 MeV/c ²	mass = 1.777 GeV/c ²	0	91.2 GeV/c ²
e electron	μ muon	τ tau	Z Z boson	
mass = 0.5 eV/c ²	mass = 1.7 MeV/c ²	mass = 1.6 MeV/c ²	80.4 GeV/c ²	
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	



General Relativity

- Einstein 1915: **General Relativity**:



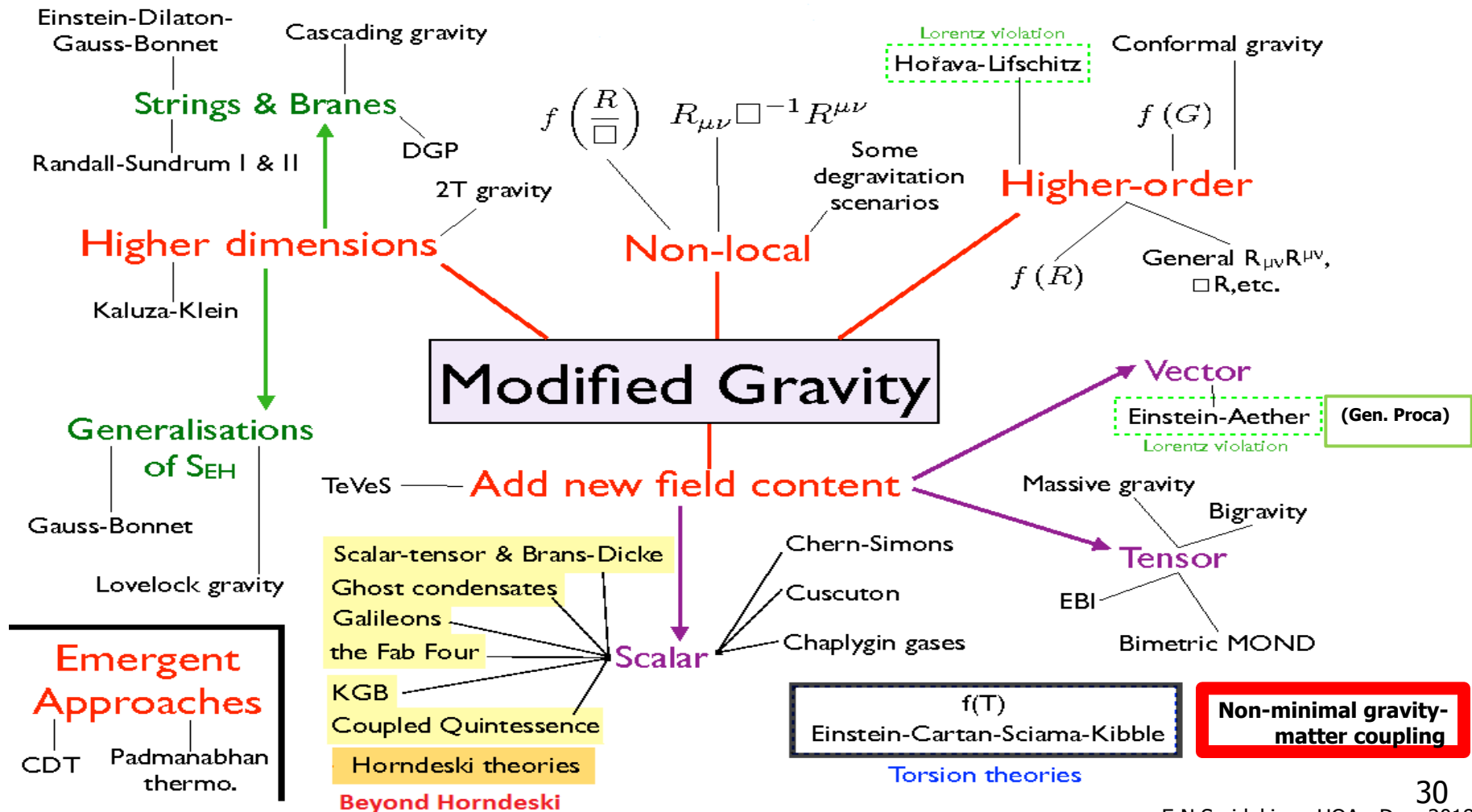
energy-momentum source of spacetime **Curvature**

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - 2\Lambda] + \int d^4x L_m(g_{\mu\nu}, \psi)$$

$$\Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}$$

$$\text{with } T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta L_m}{\delta g_{\mu\nu}}$$

Modified Gravity





Inflation: scalar field

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi)$$

$$\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad P = \frac{1}{2} \dot{\phi}^2 - V(\phi),$$

$$H^2 = \frac{8\pi}{3m_{\text{pl}}^2} \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) \right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi(\phi) = 0$$

Inflation: scalar field

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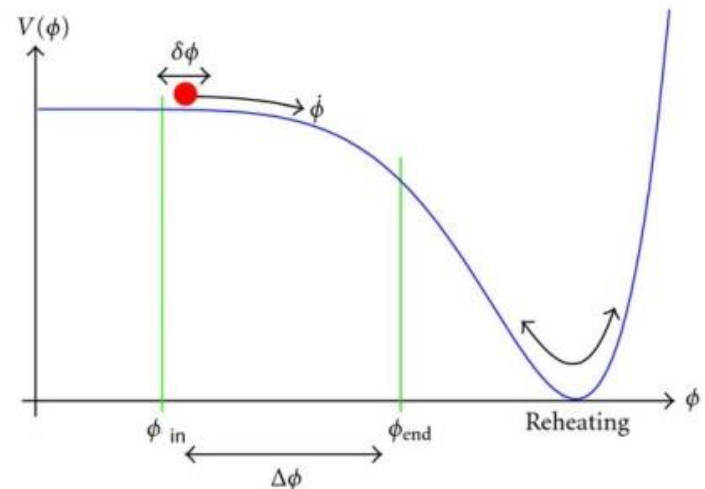
$$\ddot{\phi} + 3H\dot{\phi} + V_\phi(\phi) = 0$$

- **Slow-roll conditions:** $\dot{\phi}^2/2 \ll V(\phi)$ and $|\ddot{\phi}| \ll 3H|\dot{\phi}|$

$$H^2 \simeq \frac{8\pi V(\phi)}{3m_{\text{pl}}^2},$$

$$3H\dot{\phi} \simeq -V_\phi(\phi)$$

$$N \equiv \ln \frac{a_f}{a} = \int_t^{t_f} H dt \simeq \frac{8\pi}{m_{\text{pl}}^2} \int_{\phi_f}^{\phi} \frac{V}{V_\phi} d\phi$$

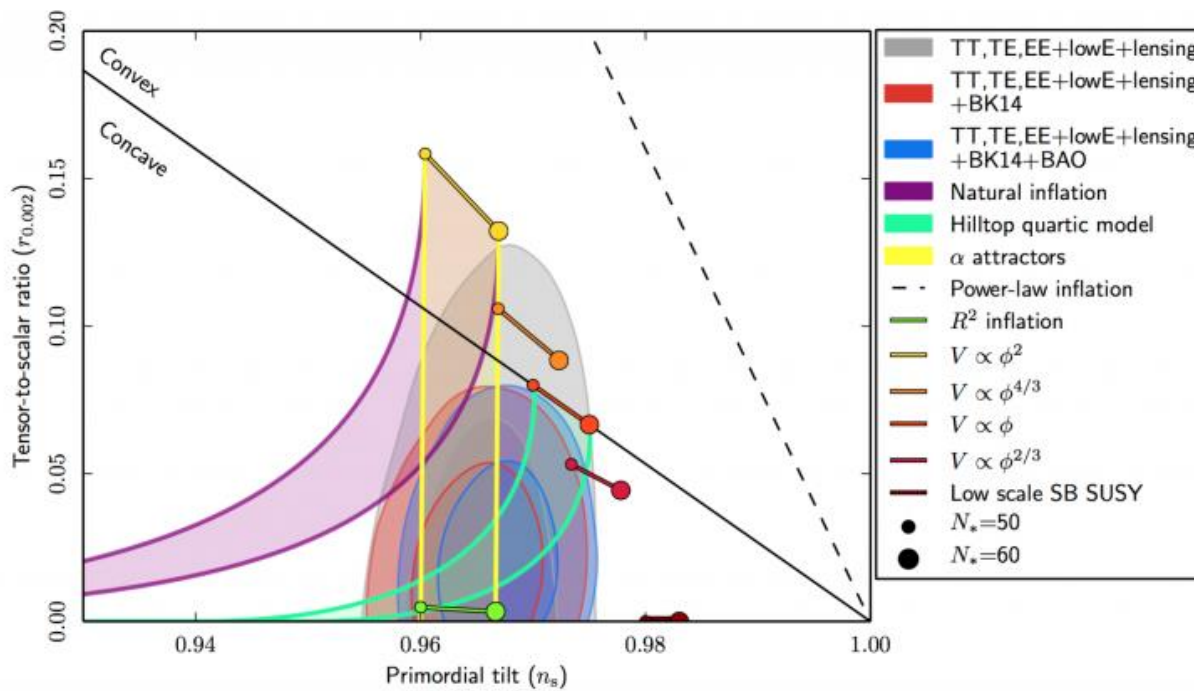


Inflation: scalar field

$$\epsilon = \frac{m_{\text{pl}}^2}{16\pi} \left(\frac{V_\phi}{V} \right)^2, \quad \eta = \frac{m_{\text{pl}}^2 V_{\phi\phi}}{8\pi V}, \quad \xi^2 = \frac{m_{\text{pl}}^4 V_\phi V_{\phi\phi\phi}}{64\pi^2 V^2}$$

$$n_s \approx 1 - 6\epsilon + 2\eta$$

$$r \approx 16\epsilon$$





Scalar-Tensor Theories

- Most general 4D scalar-tensor theories having second-order field equations:

$$L_H = \sum_{i=2}^5 L_i$$

$$L_2[K] = K(\phi, X)$$

$$L_3[G_3] = -G_3(\phi, X) \diamond \phi$$

$$L_4[G_4] = G_4(\phi, X)R + G_{4,X} [(\diamond \phi)^2 - (\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi)]$$

$$L_5[G_5] = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu \nabla^\nu \phi) - \frac{1}{6}G_{5,X} [(\diamond \phi)^3 - 3(\diamond \phi)(\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) + 2(\nabla^\mu \nabla_\alpha \phi)(\nabla^\alpha \nabla_\beta \phi)(\nabla^\beta \nabla_\mu \phi)]$$

$$X = -\partial^\mu \phi \partial_\mu \phi / 2$$

[G. Horndeski, Int. J. Theor. Phys. 10]

Horndeski Theories

- Most general 4D scalar-tensor theories having second-order field equations:

$$L_H = \sum_{i=2}^5 L_i$$

$$L_2[K] = K(\phi, X)$$

$$L_3[G_3] = -G_3(\phi, X) \diamond \phi$$

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$$L_5[G_5] = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu \nabla^\nu \phi) - \frac{1}{6}G_{5,X} [(\diamond \phi)^3 - 3(\diamond \phi)(\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) + 2(\nabla^\mu \nabla_\alpha \phi)(\nabla^\alpha \nabla_\beta \phi)(\nabla^\beta \nabla_\mu \phi)]$$

$$X = -\partial^\mu \phi \partial_\mu \phi / 2$$

[G. Horndeski, Int. J. Theor. Phys. 10]



- Coincides with Generalized Galileon theories

$$\phi \rightarrow \phi + c, \quad \partial_\mu \phi \rightarrow \partial_\mu \phi + b_\mu$$

[Nicolis, Rattazzi, Trincherini, PRD 79]

Horndeski Cosmology (background)

Field Equations: $L.H.S = R.H.S$

In flat FRW:

$$2XK_{,X} - K + 6X\dot{\phi}HG_{3,X} - 2XG_{3,\phi} - 6H^2G_4 + 24H^2X(G_{4,X} + XG_{4,XX}) - 12HX\dot{\phi}G_{4,\phi X} - 6H\dot{\phi}G_{4,\phi} + 2H^3X\dot{\phi}(5G_{5,X} + 2XG_{5,XX}) - 6H^2X(3G_{5,\phi} + 2XG_{5,\phi X}) = -\rho_m$$

$$K - 2X(G_{3,\phi} + \ddot{\phi}G_{3,X}) + 2(3H^2 + 2\dot{H})G_4 - 12H^2XG_{4,X} - 4HX\dot{X}G_{4,X} - 8\dot{H}XG_{4,X} - 8HX\dot{X}G_{4,XX} + 2(\ddot{\phi} + 2H\dot{\phi})G_{4,\phi} + 4XG_{4,\phi\phi} + 4X(\ddot{\phi} - 2H\dot{\phi})G_{4,\phi X} - 2X(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi})G_{5,X} - 4H^2X^2\ddot{\phi}G_{5,XX} + 4HX(\dot{X} - HX)G_{5,\phi X} + 2[2(\dot{H}X + H\dot{X}) + 3H^2X]G_{5,\phi} + 4HX\dot{\phi}G_{5,\phi\phi} = -p_m$$

$$\frac{1}{a^3} \frac{d}{dt} (a^3 J) = P_\phi$$

with $J = \dot{\phi}K_{,X} + 6HXG_{3,X} - 2\dot{\phi}G_{3,\phi} + 6H^2\dot{\phi}(G_{4,X} + 2XG_{4,XX}) - 12HXG_{4,\phi X} + 2H^3X(3G_{5,X} + 2XG_{5,XX}) - 6H^2\dot{\phi}(G_{5,\phi} + XG_{5,\phi X})$

$$P_\phi = K_{,\phi} - 2X(G_{3,\phi\phi} + \ddot{\phi}G_{3,\phi X}) + 6(2H^2 + \dot{H})G_{4,\phi} + 6H(\dot{X} + 2HX)G_{4,\phi X} - 6H^2XG_{5,\phi\phi} + 2H^3X\dot{\phi}G_{5,\phi X}$$

[De Felice, Tsujikawa JCAP 1202]

Horndeski Cosmology (perturbations)

■ **Scalar perturbations:** $ds^2 = -(1 + 2\psi)dt^2 + a^2(1 - 2\phi)\delta_{ij}dx^i dx^j \quad \Rightarrow L.H.S = R.H.S$

■ **No-ghost condition:** $Q_s \equiv \frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2} > 0$

■ **No Laplacian instabilities condition:** $c_s^2 \equiv \frac{3(2w_1^2w_2H - 4w_2^2w_4 + 4w_1w_2\dot{w}_1 - 2w_1^2\dot{w}_2) - 6w_1^2(\rho_m + p_m)}{w_1(4w_1w_3 + 9w_2^2)} > 0$

with $w_1 \equiv 2(G_4 - 2XG_{4,X}) - 2X(G_{5,X}\dot{\phi}H - G_{5,\phi})$

$$w_2 \equiv -2G_{3,X}X\dot{\phi} + 4G_4H - 16X^2G_{4,XX}H + 4(\dot{\phi}G_{4,\phi X} - 4HG_{4,X})X + 2G_{4,\phi}\dot{\phi} \\ + 8X^2G_{5,\phi X}H + 2HX(6G_{5,\phi} - 5HG_{5,X}\dot{\phi}) - 4G_{5,XX}\dot{\phi}X^2H^2$$

$$w_3 \equiv 3X(K_{,X} + 2XK_{,XX}) + 6X(3X\dot{\phi}HG_{3,XX} - G_{3,\phi X}X - G_{3,\phi} + 6\dot{\phi}HG_{3,X}) \\ + 18H(4HX^3G_{4,XXX} - HG_4 - 5X\dot{\phi}G_{4,\phi X} - G_{4,\phi}\dot{\phi} + 7HG_{4,X}X + 16HX^2G_{4,XX} - 2X^2\dot{\phi}G_{4,X\phi X}) \\ + 6H^2X(2H\dot{\phi}G_{5,XXX}X^2 - 6X^2G_{5,\phi XX} + 13XH\dot{\phi}G_{5,XX} - 27G_{5,\phi X}X + 15H\dot{\phi}G_{5,X} - 18G_{5,\phi})$$

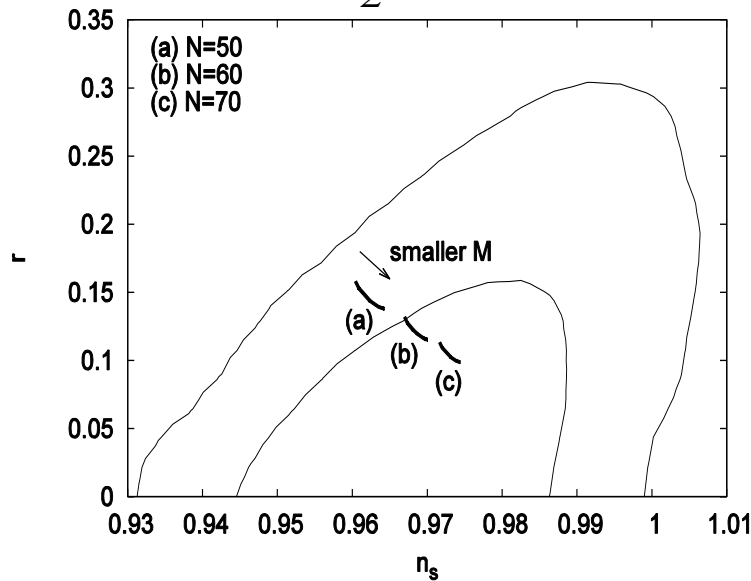
$$w_4 \equiv 2G_4 - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}$$

[De Felice, Tsujikawa JCAP 1202]

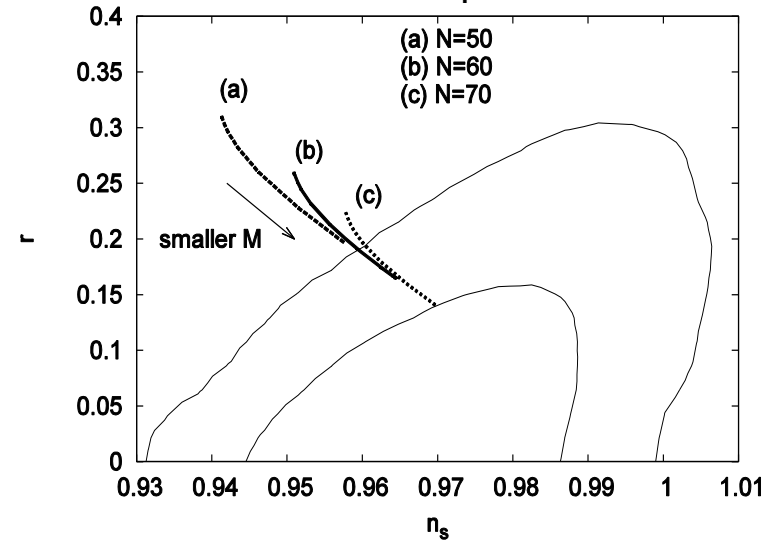
Inflation in Horndeski Theories

$K(\phi, X) = X - V(\phi), \quad G_3(\phi, X) = \frac{c_3}{M^3} X, \quad G_4 = G_5 = 0$
[Ohashi, Tsujikawa, JCAP 1210]

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$



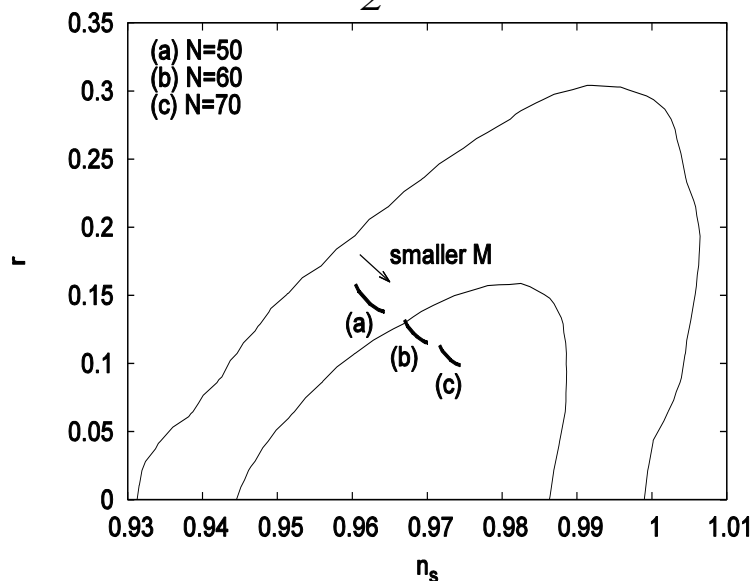
$$V(\phi) = \frac{1}{4} \lambda \phi^4$$



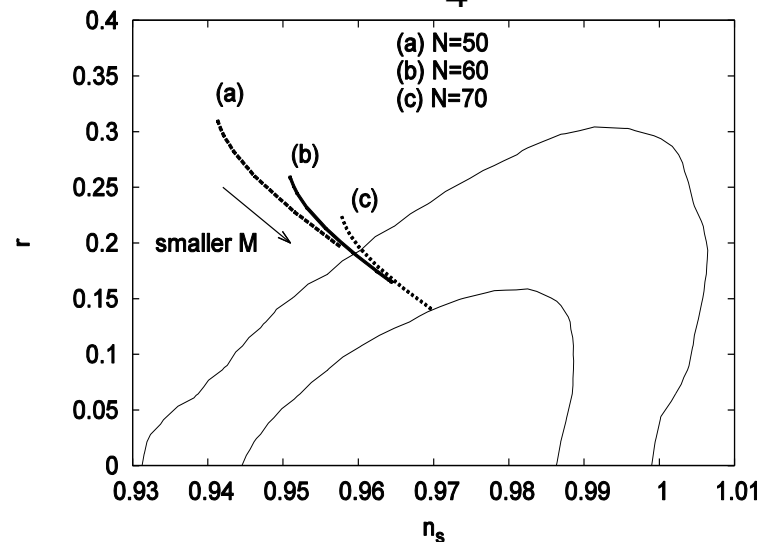
Inflation in Horndeski Theories

■ $K(\phi, X) = X - V(\phi)$, $G_3(\phi, X) = \frac{c_3}{M^3} X$, $G_4 = G_5 = 0$ [Ohashi, Tsujikawa, JCAP 1210]

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$



$$V(\phi) = \frac{1}{4} \lambda \phi^4$$



■ **G-Inflation (Shift-symmetric):** $K(\phi, X) = X + \frac{X^2}{2M^3\mu}$, $G_3(\phi, X) = \frac{1}{M^3} X$, $G_4 = G_5 = 0$

$$r \approx 0.17$$

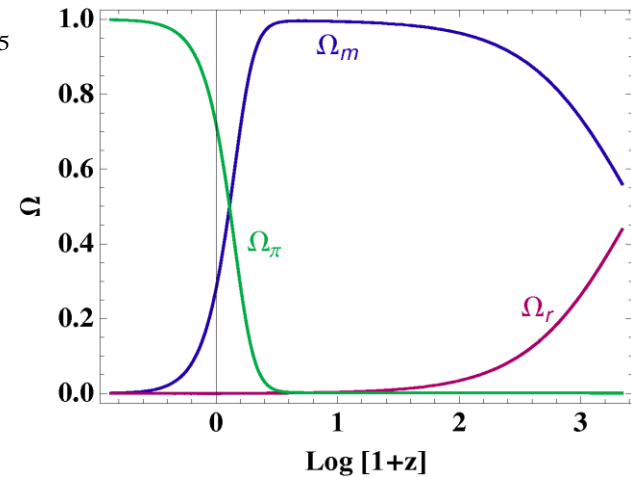
[Kobayashi, Yamaguchi, Yokoyama PRL 105]

[Banerjee, Saridakis PRD 95]

Dark Energy in Horndeski Theories

- $K(\phi, X) = c_2 X$, $G_3(\phi, X) = c_3$, $G_4 = 1$, $G_5 = c_5$
- Background evolution: Universe thermal history

[Ali,Gannouji,Sami PRD 82] [Leon, Saridakis JCAP 1303]

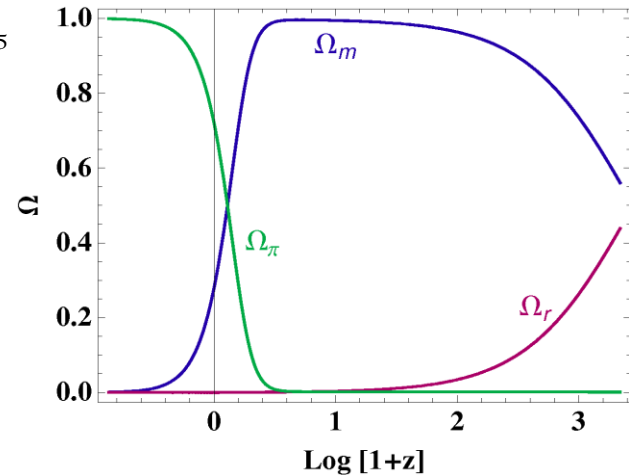


Dark Energy in Horndeski Theories

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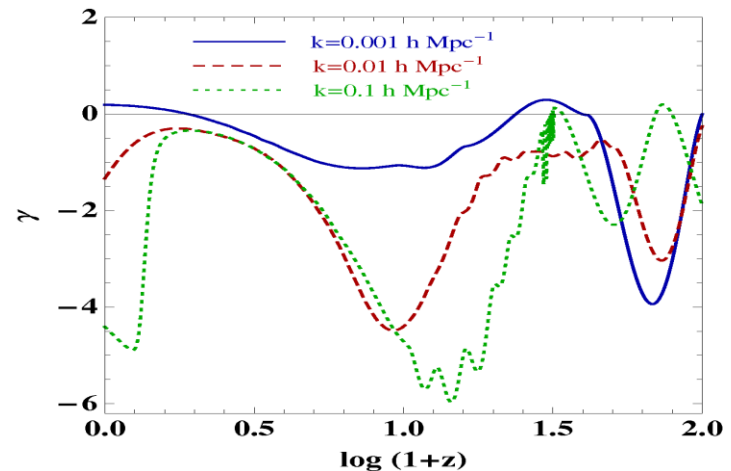
[Leon, Saridakis JCAP 1303]



- Perturbations: $\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{\text{eff}} \rho_m \delta_m$
 with $G_{\text{eff}} = G_{\text{eff}}(\phi, K, G_3, G_4, G_5)$

- Clustering growth rate: $\frac{d \ln \delta_m}{d \ln a} = \Omega_m^\gamma(a)$

$\gamma(z)$: Growth index.



[Ali,Gannouji,Sami PRD 82]

Nonminimal Derivative Coupling – Dark Energy

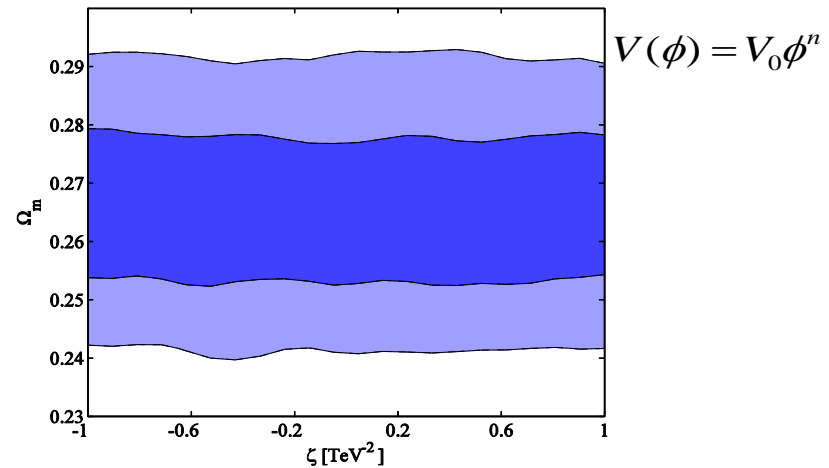
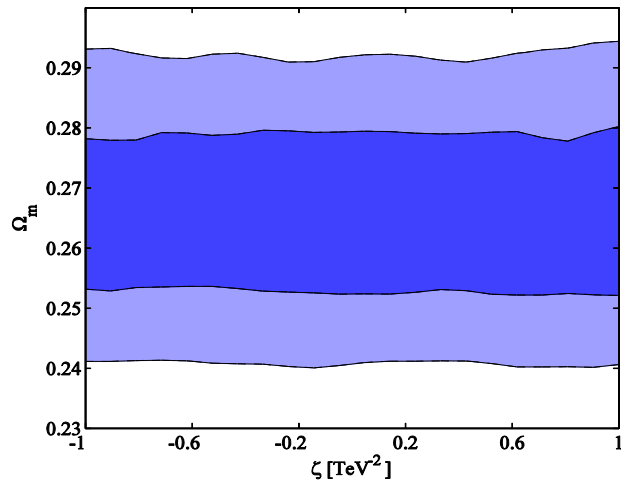
$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \frac{1}{2} (g_{\mu\nu} - \zeta G_{\mu\nu}) \partial^\mu \phi \partial^\nu \phi - V(\phi) \right] + S_m + S_r$$

- In flat FRW:

$$H^2 = \frac{8\pi G}{3} \left[\frac{\dot{\phi}^2}{2} (1 + 9\zeta H^2) + V(\phi) + \rho_m + \rho_r \right]$$

$$2\dot{H} + 3H^2 = -8\pi G \left[\frac{\dot{\phi}^2}{2} \left[1 - \zeta \left(2\dot{H} + 3H^2 + \frac{4H\ddot{\phi}}{\dot{\phi}} \right) \right] - V(\phi) + p_m + p_r \right]$$

[Saridakis, Suskov PRD 81]



[Dent, Dutta, Saridakis, Xia JCAP 1311]



Dark Matter – Dark Energy Interaction

- **Theoretical argument:** In principle, since **the underlying theory** and the microphysics of both **dark energy** and **dark matter** is **unknown**, possible mutual **interactions** cannot be excluded.



Dark Matter – Dark Energy Interaction

- **Theoretical argument:** In principle, since **the underlying theory** and the microphysics of both **dark energy** and **dark matter** is **unknown**, possible mutual **interactions** cannot be excluded.
- **Phenomenological argument:** Alleviate the **coincidence problem**: Why are the **DE and DM densities nearly equal** today, although they **scale independently** through the expansion history

[Billyard, Coley, PRD 61]

[Mimoso, Nunes, Pavon, PRD 73]

[Chen, Gong, Saridakis JCAP 0904]



DM – DE Interaction

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R \right] + S_\phi + S_{DM} + S_b$$

- Assume that **DE** and **DM** are effectively described by **perfect fluids**.

$$H^2 = \frac{8\pi G}{3} (\rho_{DE} + \rho_{DM})$$

$$\dot{H} = -4\pi G (\rho_{DE} + p_{DM})$$



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$$H^2 = \frac{8\pi G}{3} (\rho_{DE} + \rho_{DM})$$

$$\dot{H} = -4\pi G (\rho_{DE} + p_{DM})$$

- Equations give only the **total conservation**, namely

$$\nabla^b T_{ab}^{(tot)} = \nabla^b [T_{ab}^{(DE)} + T_{ab}^{(DM)}] = 0$$

- If we assume DM conservation, i.e. $\nabla^b T_{ab}^{(DM)} = 0$ then DE is also conserved: $\nabla^b T_{ab}^{(DE)} = 0$

$$\Rightarrow \dot{\rho}_{DM} + 3H(\rho_{DM} + p_{DM}) = 0$$

$$\Rightarrow \dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0$$



DM – DE Interaction

- However, it is not forbidden to assume **DM – DE interaction** by arbitrarily splitting as:

$$\nabla^b T_{ab}^{(DM)} = Q_a$$

$$\nabla^b T_{ab}^{(DE)} = -Q_a$$

with Q_a a **phenomenological descriptor** of the interaction (positive Q_a corresponds to **energy transfer** from DE to DM and vice versa).



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with Q_a a **phenomenological descriptor** of the interaction (positive corresponds to **energy transfer** from DE to DM and vice versa).

- Despite **possible pathologies** (curvature perturbation blowing up in super-Hubble scales [Valiviita,Majerotto,Maartens, JCAP 0807]) it leads **to interesting cosmological behavior**.

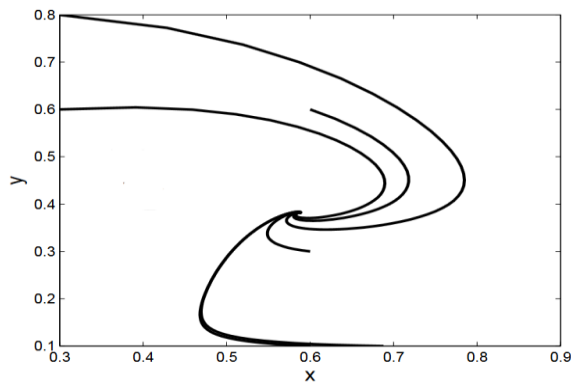


Phenomenological Models

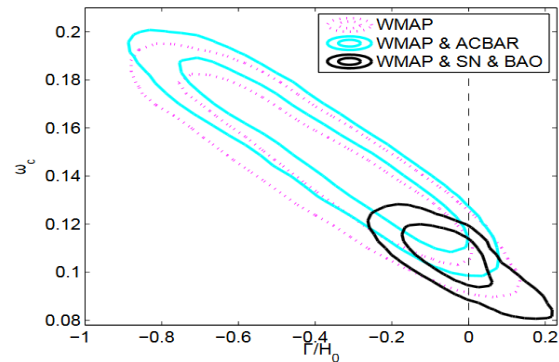
- I) $Q = Q_0 = 3H(\alpha_{DE}\rho_{DE} + \alpha_{DM}\rho_{DM})$
- II) $Q = Q_0 = \Gamma\rho_{DM}$
- III) $Q = Q_0 = \alpha\kappa^{2n}H^{3-2n}\rho_{DM}^n$
- etc...

Phenomenological Models

- I) $Q = Q_0 = 3H(\alpha_{DE}\rho_{DE} + \alpha_{DM}\rho_{DM})$
- II) $Q = Q_0 = \Gamma\rho_{DM}$
- III) $Q = Q_0 = \alpha\kappa^{2n}H^{3-2n}\rho_{DM}^n$
- etc...
- Obtain **late time attractors** with $R \equiv \rho_{DE} / \rho_{DM} \sim 1$



[Chen, Gong, Saridakis JCAP 0904]



[Valiviita, Majerotto, Maartens, MNRAS 402]

[Caldera-Cabral, Maartens, Urena-Lopez, PRD 79]

Another approach to phenomenological models

- If $Q=0$ then $\rho_{DM} = \rho_{DM0} / a^3$. Instead of imposing Q one can parametrize its effect assuming:

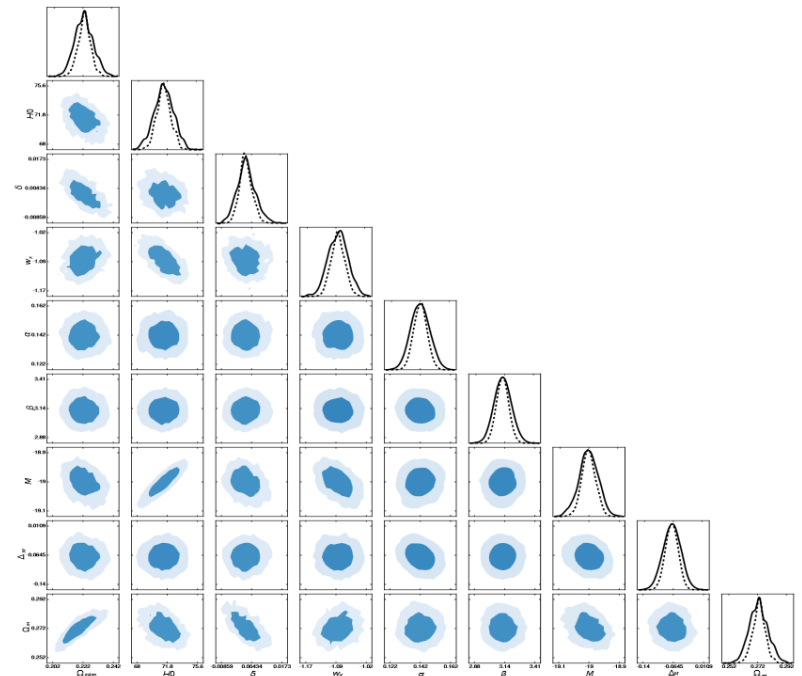
$$\rho_{DM} = \rho_{DM0} / a^{3-\delta} \quad (\text{perturbations can also be studied; obtain matter overdensity}) \quad [\text{Wang, Meng CQG 22}]$$

Param.	best-fit	mean $\pm\sigma$	95% lower	95% upper
Ω_{cdm0}	0.2246	$0.2229^{+0.0063}_{-0.0069}$	0.2099	0.2365
H_0	71.17	$71.37^{+1.3}_{-1.3}$	68.67	74.01
δ	0.00099	$0.00196^{+0.0038}_{-0.0046}$	-0.00631	0.01085
w	-1.085	$-1.087^{+0.027}_{-0.028}$	-1.139	-1.032
α	0.143	$0.1422^{+0.0065}_{-0.007}$	0.1291	0.1556
β	3.117	$3.126^{+0.079}_{-0.083}$	2.966	3.29
M	-19.04	$-19.04^{+0.041}_{-0.037}$	-19.12	-18.96
Δ_M	-0.0721	$-0.0680^{+0.024}_{-0.023}$	-0.116	-0.0211
Ω_{m0}	0.2746	$0.2729^{+0.0063}_{-0.0069}$	0.2599	0.2865

H0+SNIa+BAO+CMB

- Slight **tendency** towards **interacting DE**
 $\delta < 0$ implies energy flow DM \rightarrow DE

[Nunes, Pan, Saridakis PRD 94]





f(R) gravity

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f(R) + S_m(g_{\mu\nu}, \psi)$$

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - [\nabla_\mu \nabla_\nu - g_{\mu\nu} \diamond] f'(R) = 8\pi G T_{\mu\nu}$$

- **Field Equations** (metric formalism):

- **Conformal transformation:** $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = f'(R)g_{\mu\nu} \equiv \phi g_{\mu\nu}, \quad d\phi = \sqrt{\frac{2\omega_0 + 3}{16\pi G}} \frac{d\phi}{\phi}$

$$\Rightarrow_{\omega_0=0} S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{\tilde{R}}{16\pi G} - \frac{1}{2} \partial^\alpha \phi \partial_\alpha \phi - U(\phi) \right] + S_m \left(e^{-\sqrt{16\pi G/3}} \tilde{g}_{\mu\nu}, \psi \right) \quad U(\phi) = \frac{Rf'(R) - f(R)}{16\pi G [f'(R)]^2}$$



f(R) cosmology - Inflation

■ **Friedmann Equations** (metric formalism):

$$3FH^2 = \frac{FR - f}{2} - 3H\dot{F} + 8\pi G \rho_m$$
$$-2F\dot{H} = \ddot{F} - H\dot{F} + 8\pi G(\rho_m + p_m)$$

$F(R) \equiv f'(R)$
 $R = 12H^2 + 6\dot{H}$

f(R) cosmology - Inflation

Friedmann Equations (metric formalism):

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$$-2F\dot{H} = \ddot{F} - H\dot{F} + 8\pi G(\rho_m + p_m)$$

$$F(R) \equiv f'(R)$$

$$R = 12H^2 + 6\dot{H}$$

Inflation: e.g. Starobinsky inflation

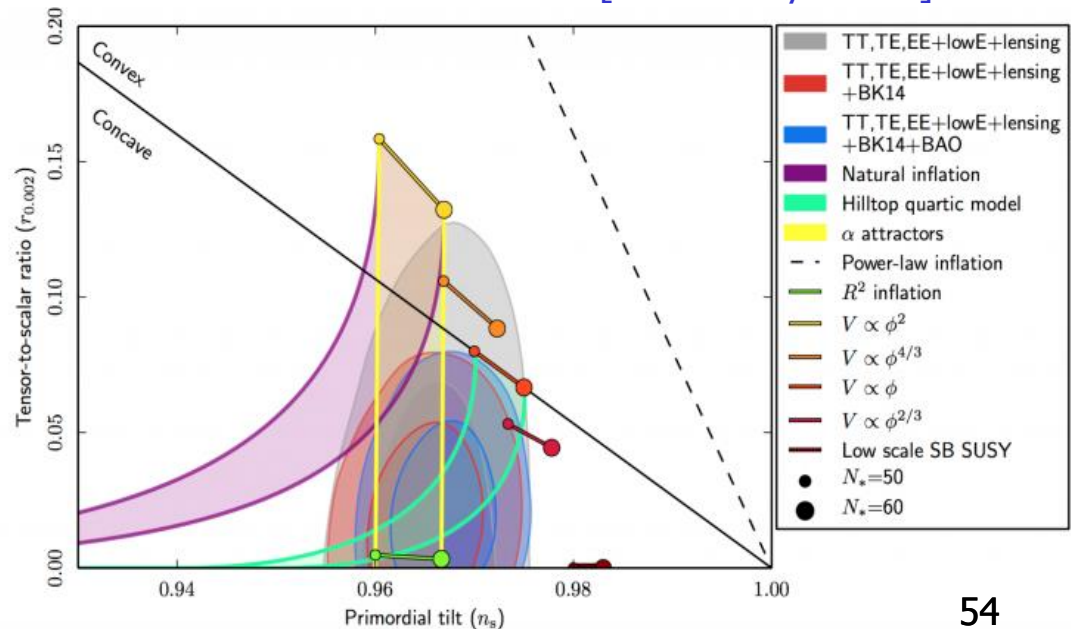
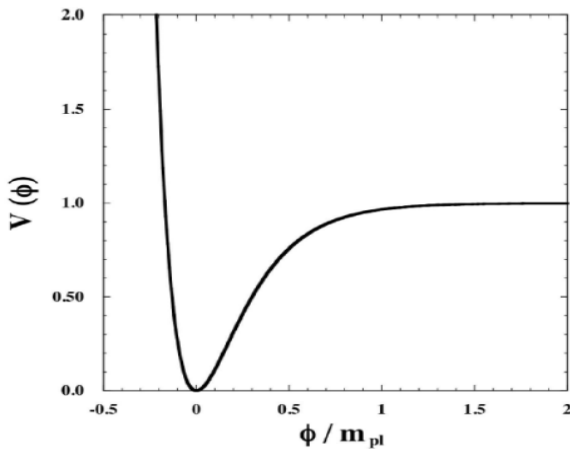
$$f(R) = R + \frac{R^2}{6M^2}$$

$$\Rightarrow V(\phi) = \frac{3M^2}{32\pi G} \left(1 - e^{-\sqrt{2/3}8\pi G\phi}\right)$$

[Starobinsky PL 91]

$$H \approx H_i - \frac{M^2}{6} (t - t_i)$$

$$T_{reh} \leq 3 \times 10^{17} g_*^{1/4} \left(\frac{M}{m_*}\right)^{3/2} GeV \quad M \approx 3 \times 10^{13} GeV$$





f(R) cosmology – Dark energy

$$8\pi G \rho_{DE} = \frac{FR - f}{2} - 3H\dot{F} + 3H^2(1 - F)$$

for **viable**: $f_{,R} > 0$, $f_{,RR} > 0$, for $R \geq R_0 (> 0)$

$$8\pi G p_{DE} = \ddot{F} + 2H\dot{F} - \frac{FR - f}{2} - (3H^2 + 2\dot{H})(1 - F)$$

[Starobinsky PLB 91]

f(R) cosmology – Dark energy

$$8\pi G \rho_{DE} = \frac{FR - f}{2} - 3H\dot{F} + 3H^2(1 - F) \quad \text{for viable: } f_{,R} > 0, f_{,RR} > 0, \text{ for } R \geq R_0 (> 0)$$

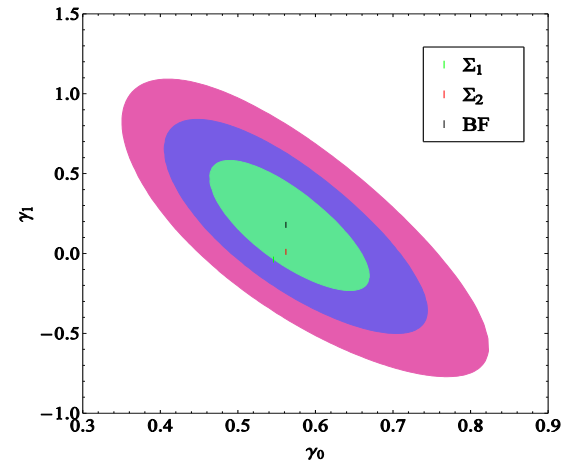
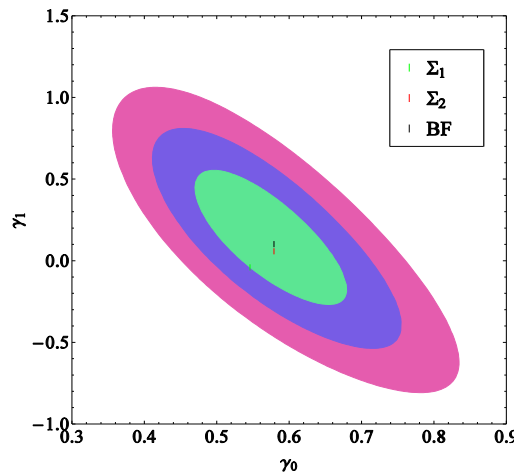
$$8\pi G p_{DE} = \ddot{F} + 2H\dot{F} - \frac{FR - f}{2} - (3H^2 + 2\dot{H})(1 - F) \quad \text{[Starobinsky PLB 91]}$$

model	$f(R)$	Constant parameters
(i) Hu-Sawicki	$R - \frac{c_1 R_{\text{HS}}(R/R_{\text{HS}})^p}{c_2(R/R_{\text{HS}})^{p+1}}$	$c_1, c_2, p(> 0), R_{\text{HS}}(> 0)$
(ii) Starobinsky	$R + \lambda R_S \left[\left(1 + \frac{R^2}{R_S^2}\right)^{-n} - 1 \right]$	$\lambda(> 0), n(> 0), R_S$
(iii) Tsujikawa	$R - \mu R_T \tanh\left(\frac{R}{R_T}\right)$	$\mu(> 0), R_T(> 0)$
(iv) Exponential	$R - \beta R_E (1 - e^{-R/R_E})$	β, R_E

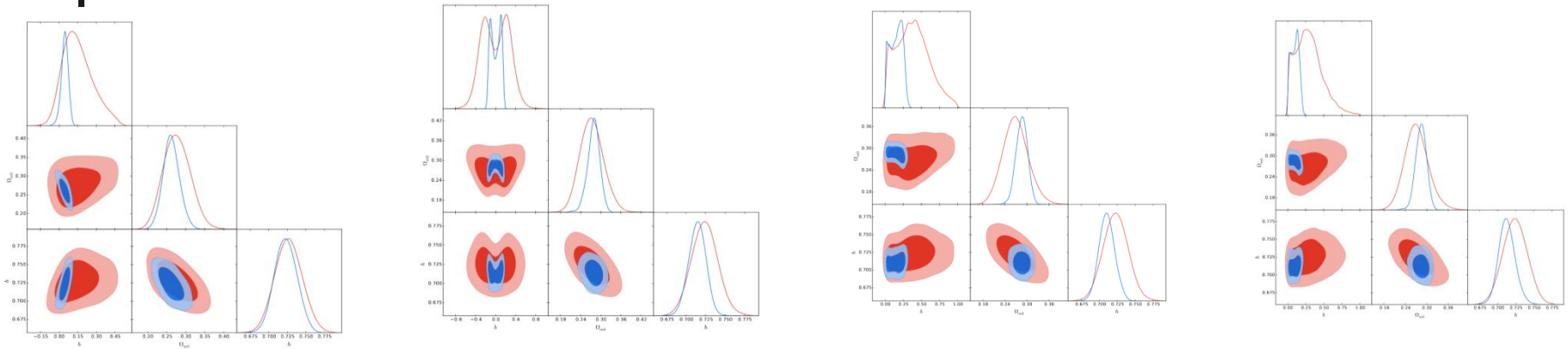
[Bamba,Geng,Lee JCAP 1011]

$$G_{\text{eff}} = \frac{G}{f'} \frac{1 + 4 \frac{k^2}{a^2} \frac{f''}{f'}}{1 + 3 \frac{k^2}{a^2} \frac{f''}{f'}}$$

$$\frac{d \ln \sigma_m}{d \ln a} = \epsilon_m \chi_m$$



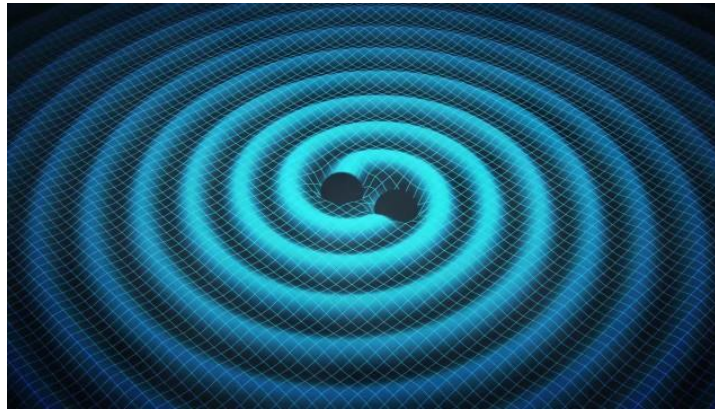
f(R) cosmology – Dark energy



Models	CC+ H_0				JLA + BAO + CC + H_0			
	AIC	ΔAIC	BIC	ΔBIC	AIC	ΔAIC	BIC	ΔBIC
Λ CDM Model	28.205	0	36.809	0	721.084	0	749.017	0
Hu-Sawicki Model	28.744	0.539	38.782	1.973	720.840	-0.244	753.428	4.411
Starobinsky Model	29.096	0.891	39.134	2.325	721.726	0.642	754.314	5.297
Tsujikawa Model	29.407	1.202	39.445	2.636	722.966	1.882	755.554	6.537
Exponential Model	29.310	1.105	39.347	2.538	722.548	1.464	755.136	6.119

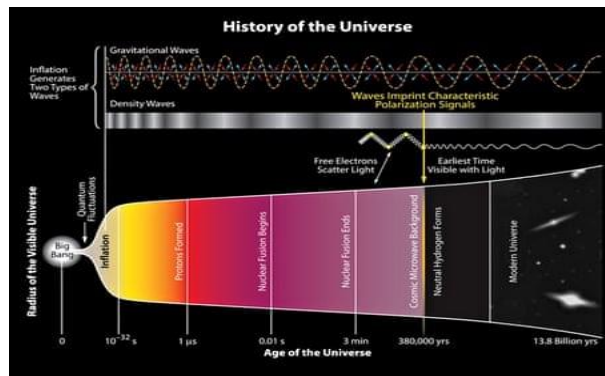
Gravitational waves

- The **GWs** are the **tensor perturbations** of the metric. Predicted in 1915, first observed in 2015. **First astronomical observation ever, not related to E/M.**
- **GWs from mergers:**



[Abbott et al, LIGO Virgo PRL 116]

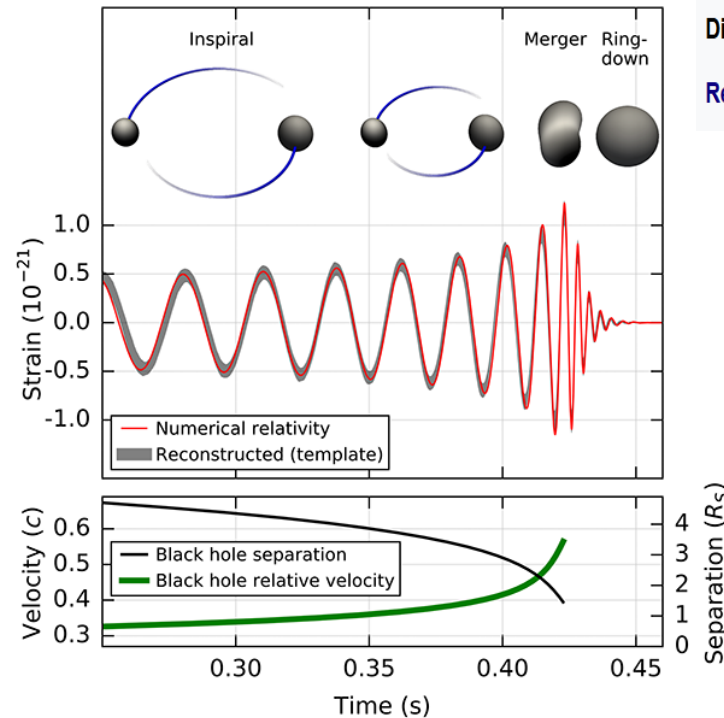
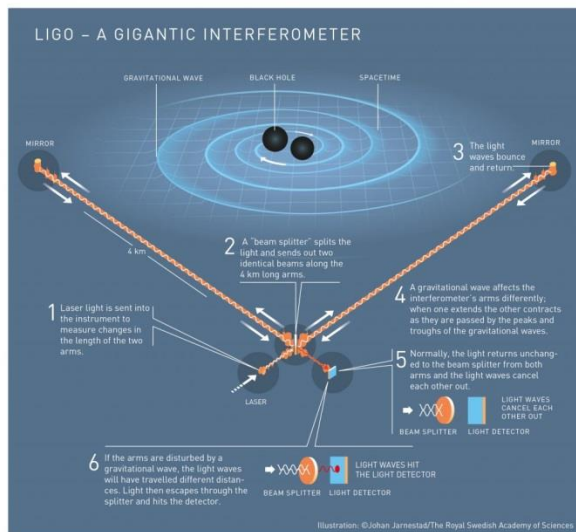
- **Primordial GWs:**



Gravitational waves

GW150914: Two **black holes** with $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$, resulting in a $62^{+4}_{-4} M_{\odot}$ black hole

Louisiana.
Washington
4km
 $10^{-18}m$



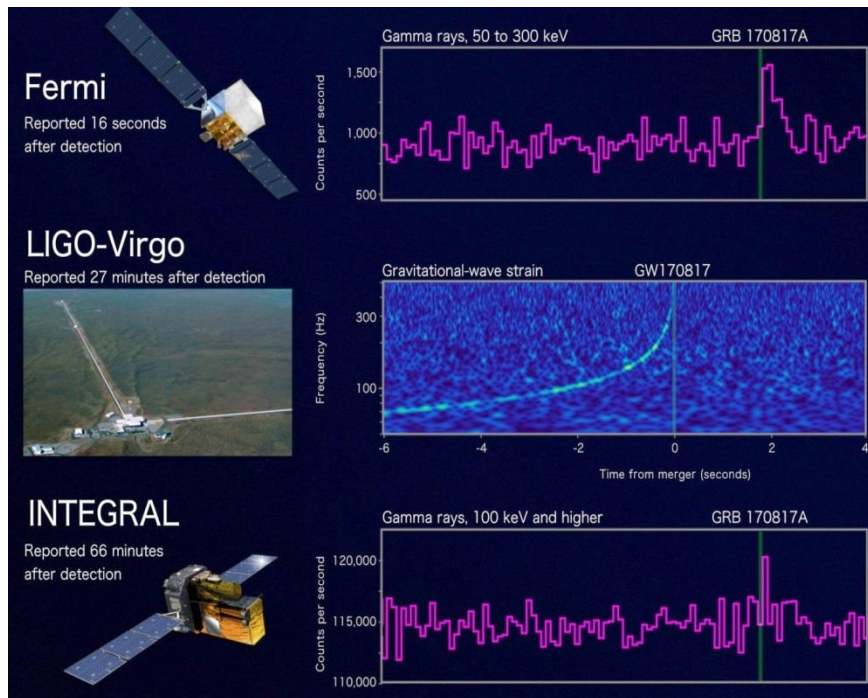
Distance	410^{+160}_{-180} Mpc
Redshift	$0.093^{+0.030}_{-0.036}$

[Abbott et al, LIGO Virgo PRL 116]

2017 Nobel Price in Physics

Gravitational waves

- **GW170817**: Two **neutron stars**, distance 40 Mpc, redshift 0.0099
- **GRB170817A**: The Electromagnetic counterpart.



[Goldstein et al, Fermi Gamma Ray Burst Monitor Astrophys.J 848]

[Abbott et al, LIGO Virgo PRL 119]

- The **era** of **multi-messenger astronomy** begins!



Gravitational waves

- In case of GWs from **black hole mergers** we know their **properties** at the **moment of detection**, and their direction (in case of three detectors). **Assuming GR and Λ CDM** we can extract their speed, distance, and properties at the **moment of emission**.



Gravitational waves

- In case of GWs from **black hole mergers** we know their **properties** at the **moment of detection**, and their direction (in case of three detectors). **Assuming GR and Λ CDM** we can extract their speed, distance, and properties at the **moment of emission**.
- In case of GWs from **neutron star mergers**, and their **E/M counterpart**, we know their **properties** at the **moment of detection** and their direction, but using the implied physics from the E/M information we can extract their speed, distance and **properties** at the **moment of emission**, **independently** of the **underlying gravitational theory and cosmological scenario**.
- **Great tool** for **testing General Relativity** and **cosmological scenarios!**

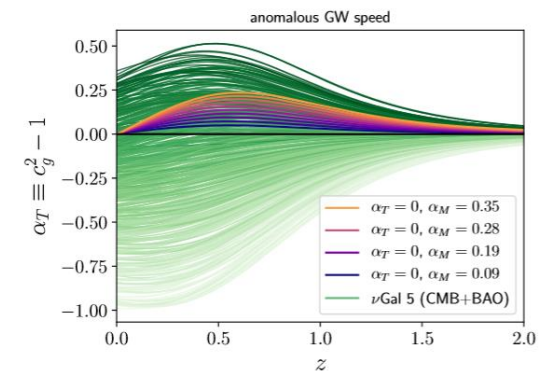
Gravitational waves

- An immediate result: **The speed of GWs is equal to the speed of light!**

GW170817 time delay $1.74 \pm 0.05\text{s}$ constrains: $-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}$

- Excludes** a large number of theories that were consistent with other data!

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [46] Brans-Dicke/ $f(R)$ [47, 48] Kinetic Gravity Braiding [50]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [49] $G_{\mu\nu}\phi^\mu\phi^\nu$ [51], $f(\phi)\cdot$ Gauss-Bonnet [52]
beyond H.	Derivative Conformal [19] [17] Disformal Tuning [21] quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817



[Ezquiaga, Zumalacarregui PRL 119]



Gravitational waves

- For tensor perturbations:

$$g_{00} = -1, \quad g_{0i} = 0,$$

$$g_{ij} = a^2 \left(\delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj} \right)$$

$$\ddot{h}_{ij} + (3 + \alpha_M) \dot{h}_{ij} + (1 + \alpha_T) \frac{k^2}{a^2} h_{ij} = 0$$

$$\alpha_M = \frac{d \log(M_*^2)}{d \log a}$$

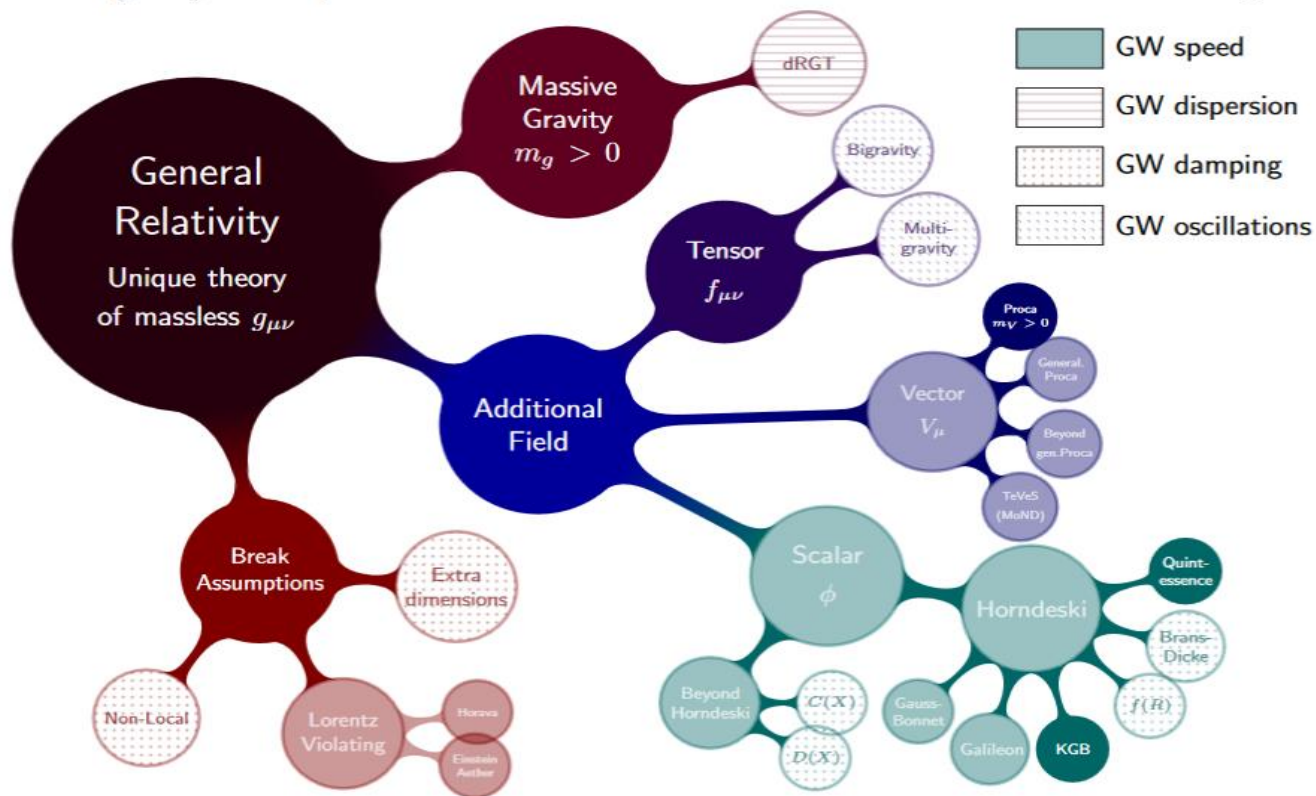
$$c_g^2 = (1 + \alpha_T)$$

- $$h_{\text{GW}} \sim h_{\text{GR}} \underbrace{e^{-\frac{1}{2} \int \nu \mathcal{H} d\eta}}_{\text{Affects amplitude}} \underbrace{e^{ik \int (\alpha_T + a^2 m^2 / k^2)^{1/2} d\eta}}_{\text{Affects phase}}$$

[Ezquiaga, Zumalacarregui 1807.09241]

Gravitational waves

Modified gravity roadmap

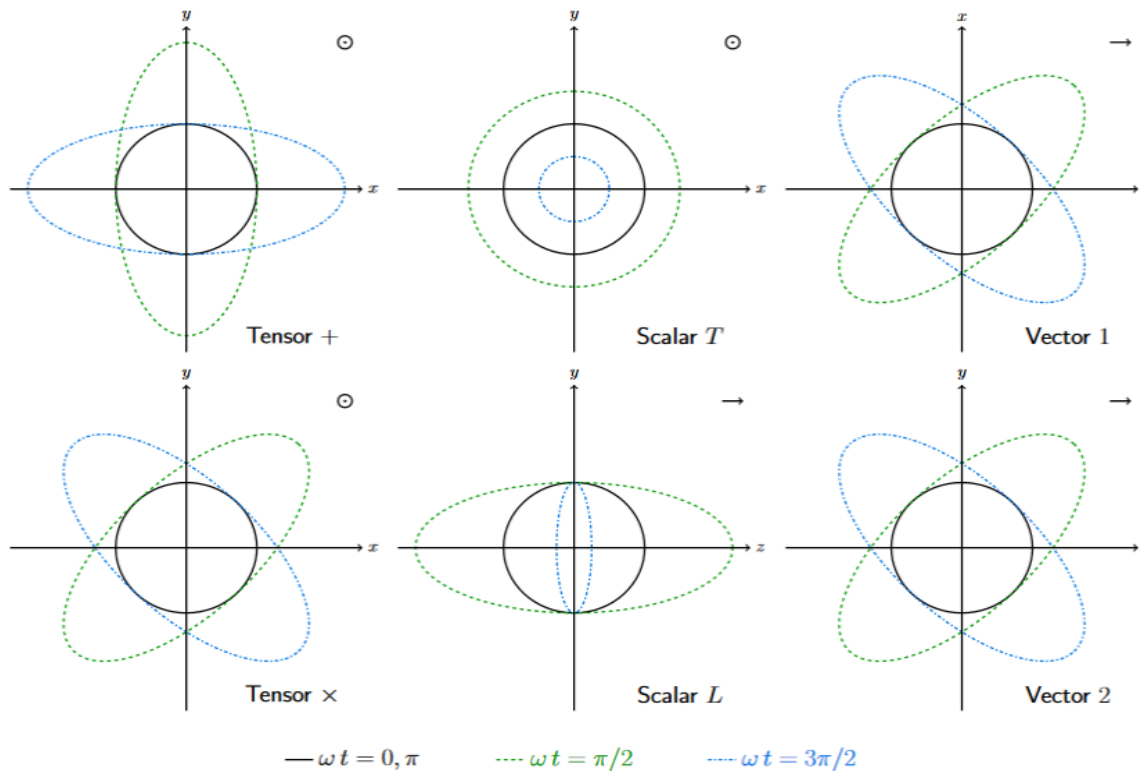


[Ezquiaga, Zumalacarregui 1807.09241]

Gravitational waves

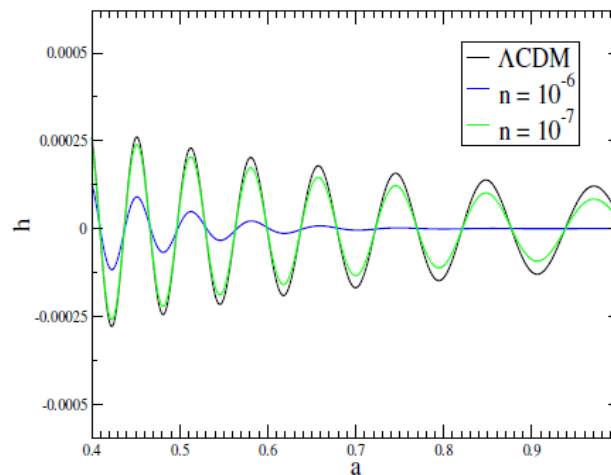
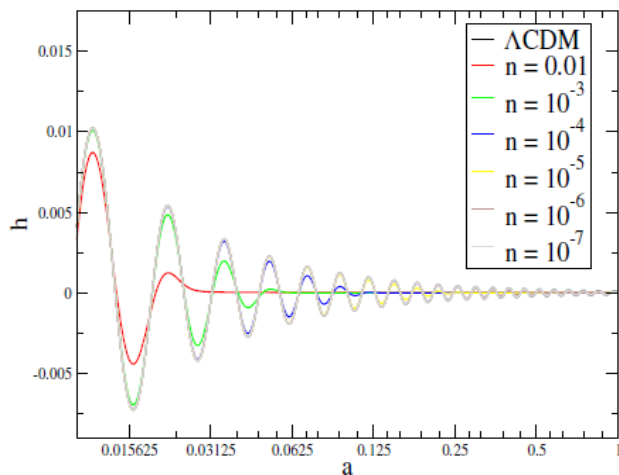
- Polarizations:

Gravitational Wave Polarizations



Gravitational waves

- Testing General Relativity, modified gravities, and various cosmological scenarios.
- The GWs properties at emission and detection are determined by them.
- Examples: $f(T)$, $f(R)$, $f(Q)$, etc



$$h_{\mu\nu}^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2\gamma_1^{(1)1} & B_1^2 \exp(ip_\mu x^\mu) & 0 \\ 0 & B_1^2 \exp(ip_\mu x^\mu) & -2\gamma_1^{(1)1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

[Cai, Li, Saridakis, Xue PRD97]

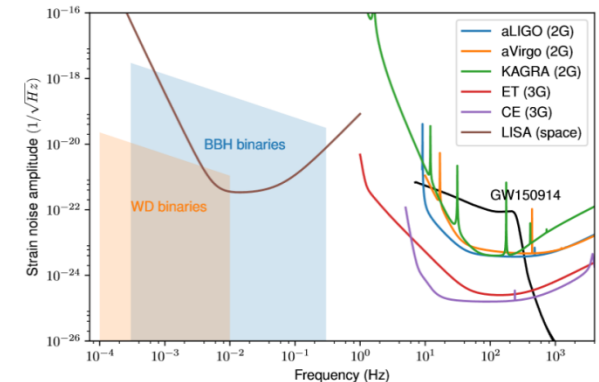
[Farrugia, Said, Gakis, Saridakis, PRD97]

[Soudi, Farrugia, Gakis, Said, Saridakis, 1810.08220]

[Nunes, Pan, Saridakis, PRD98]

Conclusions

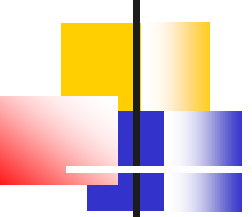
- i) The **Standard Model of Cosmology** may ask for **new physics**, definitely for **inflation** and **dark matter**, probably for **dark energy**.
- ii) We can **modify** the **Universe content**, or/and the **gravitational theory**.
- iii) We use various **observational data** (SnIa, CMB, BAO, H(z), LSS etc) in order to **constrain** the proposed **theories**.
- iv) The advancing **gravitational wave astronomy**, and especially **multi-messenger astronomy** offers a **novel tool** to test General Relativity and cosmological scenarios in **great accuracy**.
- v) **A new era has begun!**





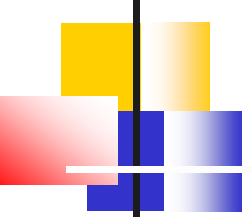
Outlook

- A **huge project** is ahead for the community:
- i) Calculate the **exact form of GWs** created from mergers in various **gravitational theories** (needs numerical gravity).
- ii) Calculate the **propagation of these GWs** from emission to detection for various **cosmological scenarios**.
- iii) Use **multi-messenger data** to test General Relativity, break degeneracies and constrain or exclude the **various theories**.
- iv) Elaborate also the creation and possible detection of **primordial GWs**.
- v) For $f(T)$ gravity, $f(R,G)$, running vacuum, higher-order theories, entropic gravity etc, **currently under investigation**
[Saridakis, Assimakis, Erices, Gakis, Palikaris, Theodosiou]
- vi) **Get prepared** for the **huge flow of data** that **will come!**

- 
- “There are the ones that **invent occult fluids** to understand the Laws of Nature. They come to conclusions, but they now run out into **dreams** and **chimeras** neglecting the **true constitutions** of the things...
However there are those that from the **simplest observation of Nature**, they reproduce **New Forces**”...

From the Preface of PRINCIPIA (II edition) 1687
by **Isaac Newton**, written by Mr. Roger Cotes.



- 
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THANK YOU!



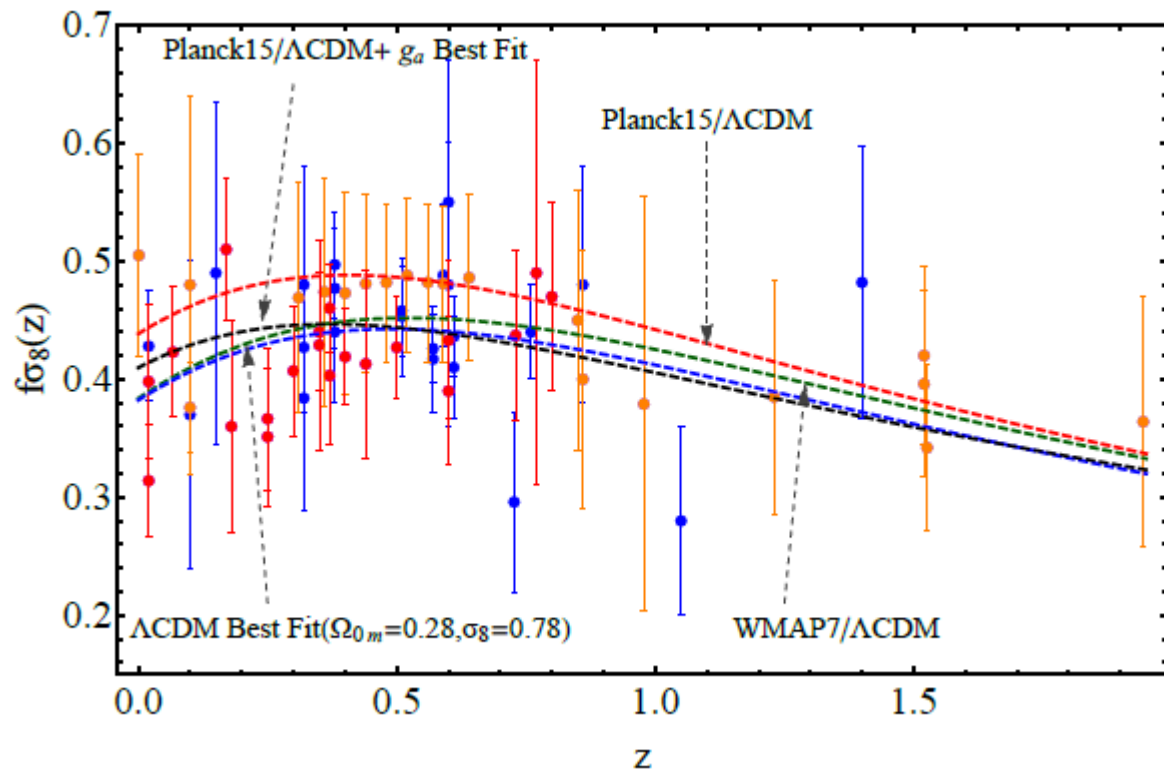




Tension1 – $f\sigma_8$

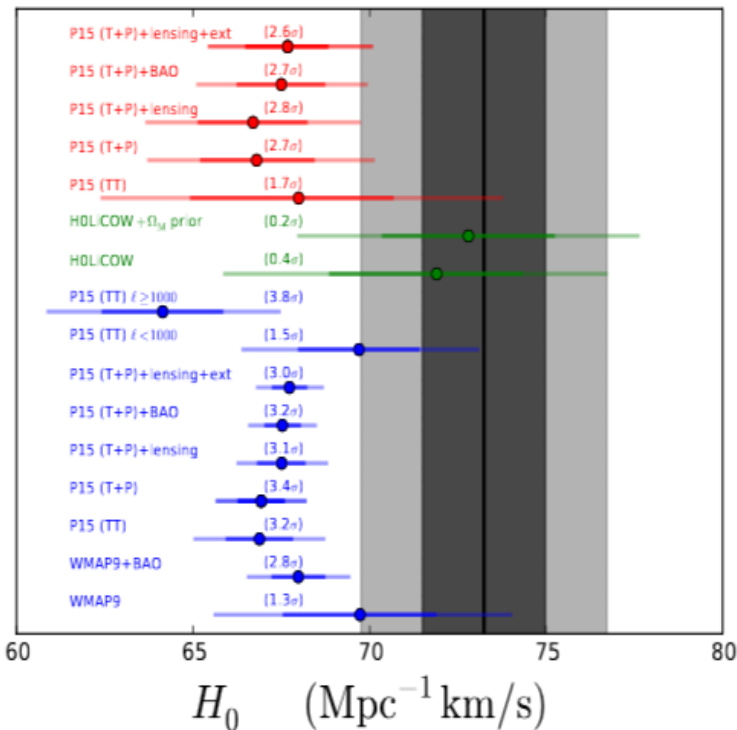
- **Tension** between the **data** and **Planck/ Λ CDM**. The data indicate a **lack of “gravitational power”** in structures on intermediate-small cosmological scales.

Parameter	Planck15/ Λ CDM [12]	WMAP7/ Λ CDM [45]
$\Omega_b h^2$	0.02225 ± 0.00016	0.02258 ± 0.00057
$\Omega_c h^2$	0.1198 ± 0.0015	0.1109 ± 0.0056
n_s	0.9645 ± 0.0049	0.963 ± 0.014
H_0	67.27 ± 0.66	71.0 ± 2.5
Ω_{0m}	0.3156 ± 0.0091	0.266 ± 0.025
w	-1	-1
σ_8	0.831 ± 0.013	0.801 ± 0.030

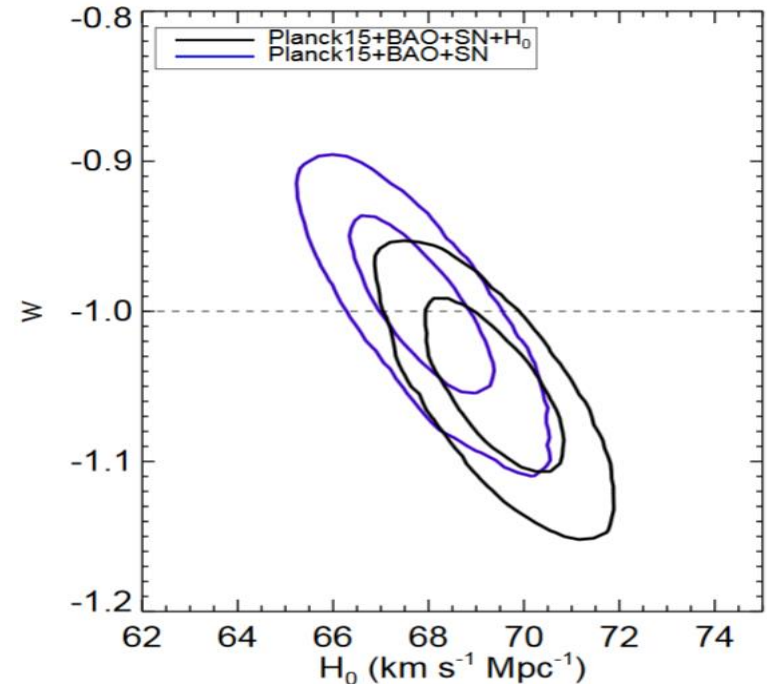


Tension2 – H0

- **Tension** between the **data** (direct measurements) and **Planck/ Λ CDM** (indirect measurements). The data indicate **a lack of "gravitational power"**.



[Bernal, Verde, Riess, JCAP1610]



[Riess et al, Astrophys.J 826]